HYBRIDGE

Distributed Control and Stochastic Analysis of Hybrid Systems Supporting Safety Critical Real-Time Systems Design

WP5: Control of uncertain hybrid systems

Model Predictive Control Formulation of Conflict Resolution Task

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July 20, 2004

Version: *0.3*
Task number: *5.2*
Deliverable number: *D5.2*
Contract: *IST-2001-32460 of European Commission*

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DOCUMENT CONTROL SHEET

Title of document:	*Model Predictive Control of Conflict Resolution Task*
Authors of document:	*A. Lecchini, J. Lygeros and D. Dimarogonas*
Deliverable number:	*D5.2*
Contract:	*IST-2001-32460 of European Comission*
Project:	*Distributed Control and Stochastic Analysis of Hybrid Systems Supporting Safety Critical Real-Time Systems Design (HYBRIDGE)*

DOCUMENT CHANGE LOG

Version #	Issue Date	Sections affected	Relevant information
0.1	25/03/04	All	First draft
0.2	25/04/04	All	First draft for internal review
0.3	20/07/04	All	Submitted to the consortium

Version 0.1		Organisation	Signature/Date
Authors	A. Lecchini	UCAM	
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	D. Dimarogonas	NTUA	

HYBRIDGE, IST-2001-32460
Work Package WP5, Deliverable D5.2

Model Predictive Control formulation of Conflict Resolution Tasks

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Abstract

We discuss a stochastic framework for air traffic conflict resolution. The conflict resolution task is posed as the problem of optimising an expected criterion. Optimisation is carried out by simulation of a Monte Carlo Markov Chain (MCMC). A numerical example illustrates the proposed strategy.

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List of Acronyms

ATC	Air Traffic Control
ATM	Air Traffic Management
BADA	Base of Aircraft Data
CD / CR	Conflict Detection / Resolution
FMS	Flight Management System
MC	Monte Carlo
MCMC	Monte Carlo Markov Chain
MPC	Model Predictive Control
PMM	Point Mass Model

1 Aim and Scope

The main aim of Deliverable D5.2 of HYBRIDGE is summarised in Task 5.2 of WP5:

> Encoding the requirements in the model predictive control framework. The problem formulation will not be the standard one. The controls enter discretely and dynamics are both hybrid and probabilistic. The MPC methodology will be extended to address these issues.

The Model Predictive Control (MPC) approach to the control of uncertain systems consists of the following steps:

1) Define a prediction model to predict the evolution of the system over a certain future horizon given the current state and future inputs.

2) Calculate the control inputs that optimise the performance of the controlled variables predicted by the model of Step 1, according to a certain criterion defined on the prediction horizon.

3) Apply the optimised control inputs to the system and collect the new state of the system when it is available, then loop to Step 2.

In this report, we formulate centralised conflict resolution in the MPC framework. In centralised conflict resolution the system under control is a set of aircraft, the controlled variables are the positions of aircrafts and the control inputs are the instructions that the aircraft receive from centralised Air Traffic Control (ATC). The performance criterion takes into account probability of conflict between aircraft and some measure of efficiency of their trajectories. In this report, we focus mainly on Steps 1 and 2 of the general procedure sketched above. This means that we will discuss a prediction model and the optimisation of inputs. An efficient recursive implementation of the optimisation, which is desirable for Step 3, is object of current work.

This document is organised as follows. In the next section we give an introduction to the conflict resolution task in a stochastic framework. In Section 3 we recall modelling of the motion of commercial aircraft in level flight. In Section 4 we discuss the choice of resolution criteria. A procedure for optimisation of expected criteria in a stochastic framework through extensive use of simulation is described in Section 5. In Section 6 we show effectiveness of the proposed algorithm in the resolution of a simple conflict. Conclusions and the discussion of future objectives are contained in Section 7.

2 Introduction

In the current organisation of Air Traffic Management the centralised Air Traffic Control (ATC) is in complete control of the air traffic and ultimately responsible for safety. Aircraft, before take off, file flight plans which cover the entire flight. During the flight, ATC sends additional instructions to them, depending on the actual traffic, in order to avoid dangerous encounters. The main objective of ATC is to maintain safe separation. The level of accepted minimum safe separation can vary with the density of the traffic and the region of airspace. Whenever possible, ATC tries also to fulfil, the, possibly conflicting, requests of aircraft and airlines (desired path to avoid turbulence, desired time of arrivals to meet schedule, etc.).

Considerable research effort has been devoted in the last decade to address the conflict detection / conflict resolution (CD/CR) problem. We define a conflict as the situation of loss of minimum safe separation between two aircraft. A largely accepted value for minimum safe separation is 5 *nmi*. In some contexts, a collision may be more appropriate to consider instead of conflicts. In a sense a collision can be thought of as a special type of conflict, with minimum separation defined to be the size of two aircraft. A valid motivation to study collisions rather than conflicts is to find out whether an estabilished minimum separation value of 5 *nmi* can be safely reduced to a lower value.

In a probabilistic CD context one has to quantify the possibility of future conflicts starting from the current position and flight plans of the aircraft and taking into account uncertainty in the future position of aircraft while they follow given nominal paths (see e.g. [8, 14, 16, 27]). In doing CD one needs a model to predict the future position of aircraft. In a probabilistic setting, the model could be either an empirical distribution of future position, or a stochastic differential equation (SDE) that describes the aircraft motion and defines implicitly a distribution for future aircraft positions (see [2, 13, 20, 21, 22, 23]). The stochastic part enters the system as the action of the wind and several uncertainties in the physics of the aircraft.

On the basis of the prediction model one can evaluate metrics related to safety. One possible metric is conflict (or collision) probability over a certain time horizon. Several methods have been developed to estimate conflict probability for a number of prediction models (see e.g. [12, 23]). Other metrics that have been considered involve the in-crossing rate which is more closely related to collision risk [2, 1]). Among other methods, Monte Carlo (MC) methods have the main advantage of allowing flexibility in the complexity of the prediction model since the model is used only as a simulator and, in principle, it is not involved in explicit calculations. In all methods a trade off exists between computational effort (simulation time in the case of MC methods) and complexity of the model. Techniques to accelerate MC methods by saving computational time are under development, using for example particle systems. The reader is referred to [15, 28] for the application of such methods to problems in ATM risk assessment.

In CR one wants to calculate suitable instructions / maneuvers to avoid a predicted conflict. A number of CR algorithms have been proposed for a deterministic setting (see e.g. [9, 16, 26]). In a stochastic setting, the research effort has concentrated mainly on CD. Few resolution strategies have been proposed in the stochastic setting, the main reason being the complexity of stochastic prediction models; simple conflict resolution maneuvers have been considered in [7, 23].

In this report we present a Monte Carlo Markov Chain (MCMC) framework for CR in a stochastic setting. The approach is borrowed from Bayesian statistics [18, 19]. We will consider a resolution criterion that takes into account separation and other factors (e.g. aircraft requests). Then, a procedure is described to estimate the resolution maneuver that optimises the expected criterion through the simulation of MCMC [24]. The interesting point in this approach is that it extends the advantages of Monte Carlo techniques to conflict resolution problems. Obviously, a possibility would be to perform Monte Carlo estimation of the expected criterion over several possible maneuvers and then select the best one. In contrast, the procedure considered in this paper performs both evaluation of the expected criterion and optimisation within the same simulation. Moreover, a grid over the space of possible maneuvers is not necessary for application of the method.

In this contribution, we restrict our attention to *level flight*. The case of level flight is meaningful from an application point of view since aircraft typically tend to fly at the same altitude most of the time. In addition, it is common ATC practice to solve conflicts between aircraft flying at the same altitude through lateral maneuvers. The approach to conflict resolution presented in this paper extends to three dimensional flight without additional theoretical issues, though more work would be needed to identify the possible actions of ATC in the three dimensional context.

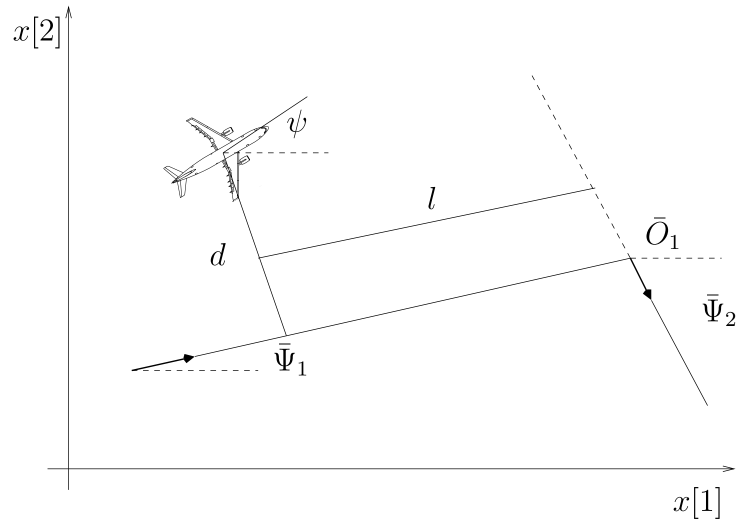

Figure 1: The flight plan

3 Modelling of aircraft motion

We recall modelling of the motion of commercial aircraft in level flight from the point of view of ATC. The model is based on description of commercial aircraft contained in the Base of Aircraft Data (BADA) database [3]. The reader is referred to [10] for a detailed presentation of the model.

Before take off, commercial aircraft file a *flight plan*, which covers the entire flight. Aircraft are equipped with a Flight Management System (FMS) that assists the pilot in following the flight plan. A flight plan (depicted in Figure 1) can be thought of a sequence of waypoints $\{\bar{O}_i\}_{i=1}^{M}$ which in the case of level flight are expressed as coordinates in a 2D reference frame. The *reference path* is the sequence of straight lines joining each waypoint to the next. Correspondingly, for each segment of the reference path, the *reference heading* is defined as the angle, $\bar{\Psi}_i = \angle[\bar{O}_i - \bar{O}_{i-1}]$ that the segment of the reference path makes with the x-axis of the frame in which the coordinates of the way points are given. Each time a waypoint is reached, the waypoint is eliminated from the flight plan and the aircraft heads to the next one according to the corresponding new reference heading. The first segment of the flight plan is therefore defined by the current reference heading $\bar{\Psi}_1$ and the first waypoint $\bar{O}_1$. In the current system the aircraft travel between waypoints with constant airspeed (i.e. speed relative to the air surrounding the aircraft) dictated by altitude dependent speed profiles which can be found in BADA.

The motion of the aircraft from the point of view of ATC is determined by the aircraft dynamics plus the action of the FMS that keeps the aircraft in track with the flight plan.

The following Point Mass Model (PMM) equations describe adequately the aircraft dynamics in level flight at the level of details needed from the point of view of ATC: x

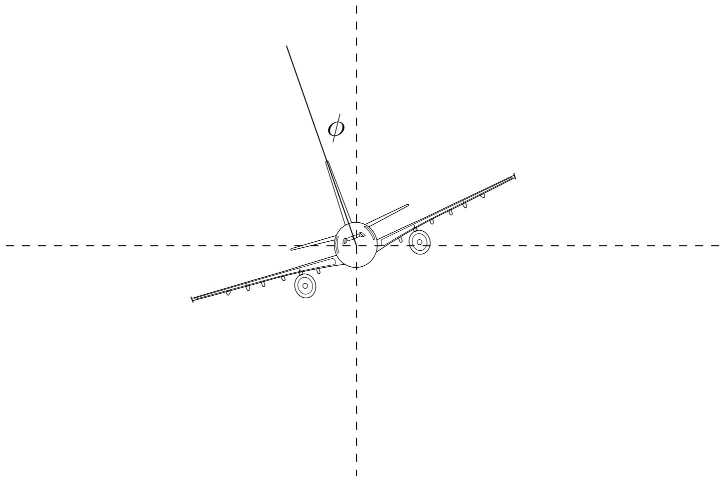

Figure 2: The bank angle ϕ

denotes the coordinates of the aircraft position in the 2D framework, ψ denotes the heading, ϕ denotes the bank angle (Figure 2), v denotes the airspeed, m denotes the mass, T denotes the engine thrust, w denotes the wind velocity. The dynamics are given by

$$\begin{bmatrix} \dot{x}[1] \\ \dot{x}[2] \\ \dot{v} \\ \dot{\psi} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} v\cos(\psi) + w[1] \\ v\sin(\psi) + w[2] \\ -\frac{C_D S \rho}{2}\frac{v^2}{m} + \frac{1}{m}T \\ \frac{g}{v}\tan(\phi) \\ -\eta T \end{bmatrix}.$$

S (surface area of the wings) , C_L (lift coefficient) and C_D (drag coefficient) are parameters that depend on the aircraft type and ρ is the air density. The motion of the aircraft is controlled through the bank angle ϕ and the thrust T.

In the model, the wind acts as an additive disturbance on the air speed. We assume that the sum of the air velocity and the wind velocity gives the ground velocity (i.e. $\dot{x}$).

In general, the wind velocity can be modelled as a random field $W(x,t)$ with space time autocorrelation (i.e. $W(x_1,t_1)$, $W(x_2,t_1)$ $W(x_1,t_2)$, $W(x_2,t_2)$ are correlated random variables).

The FMS controls the motion of the aircraft, i.e. it corrects errors with respect to the reference path and executes turns. In order to describe the action of FMS, assume that the aircraft is directed to waypoint $\bar{O}_1$ with reference heading $\bar{\Psi}_1$ and let us introduce l and d defined as

$$\begin{bmatrix} d \\ l \end{bmatrix} = \begin{bmatrix} -\sin(\bar{\Psi}_1) & \cos(\bar{\Psi}_1) \\ \cos(\bar{\Psi}_1) & \sin(\bar{\Psi}_1) \end{bmatrix} \begin{bmatrix} x[1] - \bar{O}_1[1] \\ x[2] - \bar{O}_1[2] \end{bmatrix}.$$

The moduli of l and d represents respectively the distance between the projection of the aircraft position on the nominal trajectory and the waypoint $\bar{O}_1$ and the distance of the aircraft position from the nominal trajectory. We can assume that the FMS receives as an input the error signals l, d and $\psi - \bar{\Psi}_1$ and acts on the control variables ψ and T. Several control strategies can be implemented in the FMS. A *3D FMS* regulates only the cross track error d (and possibly the heading error). The airspeed is fixed for level flight and is defined from look up tables depending on the altitude. Correspondingly, the thrust is also fixed and is obtained as the solution of the equation

$$-\frac{C_D S \rho}{2} v^2 + T = 0\,.$$

An example of a 3D FMS is illustrated in Section 6. In the case of *3.5D FMS* the waypoints are stamped with a time of arrival. The FMS regulates the error with the expected time of arrival and adjusts T to eliminate this error. In the case of a *4D FMS* the error with respect to a continuous 4D reference path (position + time) is considered.

The aircraft trajectory is then defined by the SDE describing the closed loop system consisting of the aircraft dynamics and the FMS. Let us remark that in general phenomena such as the space-time correlation of the wind field make it impossible to calculate exactly quantities such as the probability of conflict. Most of the CD methods in the literature make the approximation that the effect of the wind field can be described as a Brownian motion. Additional assumptions and approximations (such as linearisation of the dynamics about the reference path) are often needed if one requires closed form solutions for the probability of conflict. The advantage of MC methods is that, because they are simulation based, they can still be applied in the presence of nonlinear prediction models and space-time correlation of the wind field; all that is needed is to be able to develop a simulator that includes these phenomena. The disadvantage of course is that the more complicated the models get the more computationally intensive the process becomes.

4 Air Traffic Control with optimisation of an expected criterion

In our context the role of ATC is to monitor the traffic and predict possible dangerous encounters in the future. Indeed, flight plans are calculated before take off and cannot take into account the actual traffic configuration during the flight. The role of ATC is to intervene by sending suitable maneuver instructions in order to resolve predicted conflicts.

Let us consider a multiaircraft system. Without loss of generality, we assume that ATC monitors a future time horizon $[0,\, T]$ where $t = 0$ denotes the present. We model ATC instructions to each aircraft as a set of waypoints valid over the time horizon $[0,\, T]$. We denote this set of waypoints for all the aircraft as Ω. These additional waypoints can be seen as temporary modification of the original flight plans. We assume that Ω determines the nominal paths in $[0, T]$. If no ATC intervention is required then $\Omega = \bar{\Omega}$ where $\bar{\Omega}$ denotes the set of the waypoints of the original flight plans. Let us introduce also a sample time ΔT and denote X the vector of the time sequence of positions of all aircraft in $[0,\, T]$ at the sampled instants. Vector X is a random variable with probability density function $X \sim p_\Omega(x)$. The probability density $p_\Omega(x)$ is determined by the stochastic system describing the aircraft + FMS closed loop system and by the initial conditions (i.e. the positions and headings at time 0; these data are obtained as radar measurements typically every 6 seconds). The subscript Ω denotes that the distribution of X depends on the instructions received from ATC.

The objective of ATC is to select Ω in such a way that the aircraft trajectories will be conflict free and efficient. A conflict is defined as the loss of a minimum safe separation, say $\bar{c}$ (a typical value is $\bar{c} = 5$nmi), between two aircraft. If we denote $x^i(t)$ and $x^j(t)$ the positions of two different aircraft then a conflict is defined as the event

$$\exists t \in [0, T] \;:\; \|x^i(t) - x^j(t)\| < \bar{c}\,.$$

In general, for any realization of the random variable X one can define a criterion $u(\Omega, X)$ that penalises conflicting trajectories and measures the efficiency of conflict-free trajectories. Efficiency can be measured, for example, in terms of time to destination, total distance flown, or deviations of the trajectories from the reference paths. Once a criterion has been chosen, a sensible choice of Ω is then determined by the optimisation of the expected criterion

$$U(\Omega) = \int u(\Omega, x) p_\Omega(x) dx\,. \tag{1}$$

A MCMC procedure to find an approximate solution to this problem is described in the next section.

5 Simulation based optimisation

In this section we outline a simulation based procedure to optimise expected criteria. This procedure has been proposed in Bayesian statistics literature. The original idea has been presented in [18]. In [19] results on asymptotic convergence are derived.

Consider the problem of optimising the expected utility function (1) where Ω is the optimisation parameter and $p_\Omega(x)$ is a probability density function which depends on the optimisation parameter. The procedure presented below addresses the approximate optimisation of $U(\Omega)$ through extensive use of simulations. Apart from the possibility of evaluating $u(\Omega, X)$ no other particular assumptions are imposed on the optimisation criterion. Here we consider maximisation of $U(\Omega)$, i.e. $U(\Omega)$ is an expected utility. Obviously no modifications of the procedure are required in the case of minimisation of an expected cost.

The optimisation procedure relies on the definition of an augmented stochastic model in which also Ω is a random variable. The stochastic model is formed by Ω and J independent replicas of X. We denote $h(\omega, x_1, x_2, \ldots, x_J)$ the joint distribution of $(\Omega, X_1, X_2, X_3, \ldots, X_J)$. It is straightforward to see that if

$$h(\omega, x_1, x_2, \ldots, x_J) \propto \prod_j u(\omega, x_j) p_\omega(x_j) \tag{2}$$

then

$$\Omega \sim h(\omega) \propto \left[\int u(\omega, x) p_\omega(x) dx \right]^J . \tag{3}$$

This means that if we can extract from the augmented model $(\Omega, X_1, X_2, X_3, \ldots, X_J)$ then the extracted Ω's will cluster around the optimal points of $U(\Omega)$ for a sufficient high J. These extractions can be used to find an approximate solution to the original optimisation problem.

Extractions from the augmented stochastic model, with the desired joint probability density given by (2), can be obtained through a MCMC scheme. In the following algorithm $g(\omega|\bar{\omega})$ is an instrumental (or *proposal*) distribution which is freely chosen by the user. The only requirement is that $g(\omega|\bar{\omega})$ covers the support of $h(\omega)$.

MCMC ALGORITHM (METROPOLIS-HASTINGS)

Initial state $(\bar{\omega}, \bar{x}_j \, j = 1, \ldots, J)$ and $\bar{u}_J = \prod_j u(\bar{\omega}, \bar{x}_j)$

1 Extract

$$\tilde{\Omega} \sim g(\omega|\bar{\omega})$$

2 Extract

$$\tilde{X}_j \sim p_{\tilde{\Omega}}(x) \quad j = 1, \ldots, J$$

and calculate

$$\tilde{U}_J = \prod_j u(\tilde{\Omega}, \tilde{X}_j)$$

3 Extract the new state of the chain as

$$(\bar{\Omega}, \bar{U}_J) = \begin{cases} (\tilde{\Omega}, \tilde{U}_J) \text{ with probability } \rho(\bar{\omega}, \bar{u}_J, \tilde{\Omega}, \tilde{U}_J) \\ (\bar{\omega}, \bar{u}_J) \text{ with probability } 1 - \rho(\bar{\omega}, \bar{u}_J, \tilde{\Omega}, \tilde{U}_J) \end{cases}$$

where

$$\rho(\bar{\omega}, \bar{u}_J, \tilde{\omega}, \tilde{u}_J) = \min\left\{1, \frac{\tilde{u}_J}{\bar{u}_J} \frac{g(\bar{\omega}|\tilde{\omega})}{g(\tilde{\omega}|\bar{\omega})}\right\}$$

4 Repeat steps 1 through 3

The algorithm is a formulation of the Metropolis-Hasting algorithm for a desired distribution given by $h(\omega, x_1, x_2, \ldots, x_J)$ with proposal distribution given by

$$g(\omega|\bar{\omega}) \prod_j p_\omega(x_j) \,.$$

In fact, in this case, the acceptance probability for the Metropolis-Hastings algorithm is [24]

$$\frac{h(\tilde{\omega}, \tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_J)}{h(\bar{\omega}, \bar{x}_1, \bar{x}_2, \ldots, \bar{x}_J)} \frac{g(\bar{\omega}|\tilde{\omega}) \prod_j p_\omega(\bar{x}_j)}{g(\tilde{\omega}|\bar{\omega}) \prod_j p_\omega(\tilde{x}_j)} \,.$$

and by inserting (2) in the above expression one obtains exactly $\rho(\bar{\omega}, \bar{u}_J, \tilde{\omega}, \tilde{u}_J)$. The distribution of $\bar{\Omega}$ then converges to a stationary distribution given by (3) [24].

Interestingly enough, in the case in which one wants to consider a discrete version of the above MCMC then only discretisation of U_J and Ω is needed and not of X. Therefore, the algorithm complexity is independent of the dimension of the state of the problem.

6 Simulation example

In this section we illustrate the conflict resolution algorithm in a two aircraft encounter. We will first describe the model used for simulation (for a more detailed presentation of this model the reader is referred to [10]), then we will characterise the conflict and solve it by using the proposed algorithm.

We assume that the aircraft are governed by a 3D FMS which tracks the reference path and executes turns. More specifically, the following control law is assumed in the tracking of the nominal path

$$\phi = -k_1 d - k_2(\psi - \Psi) \tag{4}$$

where Ψ is the reference heading and d has been defined in Section 2. The above control law is augmented with a saturation $|\phi| \leq 35°$ in order to prevent dangerous values of the bank angle. The motion of the aircraft in following a reference path is then determined by the feedback connection of the aircraft dynamic equations of Section 2 and the control law (4).

The FMS executes turns following a smooth circular path from one reference path to the next. In order to do so the aircraft will begin tracking the next flight segment a certain distance before it reaches the next waypoint.

The wind is modelled as the sum of two components, nominal and stochastic. The nominal wind represents forecast data available to air traffic controllers. Here the nominal wind is assumed to be zero and all wind is considered to be stochastic. The stochastic wind component is modelled as a random field $W(x,t) : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}^2$. In this example we assume that the wind field is stationary and jointly Gaussian with correlation function $E[W(x_1,t_1)W(x_2,t_2)^T] = R(\Delta x, \Delta t)$, with $\Delta x = \|x_1 - x_2\|$, $\Delta t = |t_1 - t_2|$. A computationally efficient method is used for generating the random field with the required statistical structure. The wind is calculated in discrete time and only at the positions of the aircraft. The algorithm is similar to a Cholesky decomposition, but is implemented progressively because the positions at which the wind is calculated are dependent on the aircraft positions, which in turn depend on the wind the aircraft experienced at earlier times.

The model has been tuned to ensure that the trajectories it generates are realistic. In particular, in this simulation example we used:

$$R(\Delta x, \Delta t) = 7.7 e^{-6 \cdot 10^{-6} \Delta t} e^{-1.6 \cdot 10^{-6} \Delta x}$$

and

$$k_1 = 10^{-5} \qquad k_2 = 1.2\,.$$

To tune these parameters we made use of two sources of information: experimental statistics of aircraft deviations from their flight plans [20, 11, 25, 17], and a comparison of forecast wind and real wind measured from aircraft [4].

Let us now describe the two aircraft encounter. The initial configuration at time $t = 0$ is as follows, in the notation coordinates are expressed in meters. Aircraft 1 has position $x_0^1 = [-110000\ 0]$, heading $\psi_0^1 = 0$ and next waypoint $\bar{O}_1^1 = [110000\ 0]$ with reference heading $\bar{\Psi}_1^1 = 0°$. This aircraft will not change its flight plan. Aircraft 2 has position $x_0^2 = [0\ -110000]$, heading $\psi_0^2 = 90°$ and next waypoint $\bar{O}_1^2 = [0\ -100000]$ with reference heading $\bar{\Phi}_1^2 = 90°$. The second waypoint O_2^2 must be chosen in $[-100000\ 100000] \times [-100000\ 100000]$ to prevent conflict with Aircraft 1. The third and fourth waypoints of Aircraft 2 are then $\bar{O}_3^2 = [100000\ 0]$ and $\bar{O}_4^2 = [110000\ 0]$. In this case therefore the control space consists of only the position of the the second way point of aircraft 2, $\Omega = \{O_2^2\}$. Notice that the last waypoints of the two aircraft are the same. Both aircraft fly at constant airspeed $v = 150m/sec$.

We assume that the requirement for conflict resolution is that Aircraft 2 arrives after Aircraft 1 with a time separation of $300sec$.

Let us denote T_1 and T_2 the times of arrival of the two aircraft at the last waypoint [110000 0]. The following resolution criterion has then been formulated

$$u(\Omega, X, \Delta T) = \begin{cases} \varepsilon & \text{if (conflict)} \vee (T_1 > T_2) \\ \\ \varepsilon + e^{-a|\Delta T - 300|} & \text{otherwise} \end{cases}$$

where X contains the time sequence of positions of the two aircraft, $\Delta T = T_2 - T_1$, $a = 0.01$ and $\varepsilon = 0.00001$. The event conflict is defined as the loss of $5nmi = 9260m$ separation.

The MCMC algorithm of Section 5 has been employed to choose Ω that maximises the expected criterion. The proposal distribution $g(\omega|\bar{\omega})$ has been chosen as a uniform distribution $g(\omega|\bar{\omega}) = const$ with $\omega \in [-100000\ 100000] \times [-100000\ 100000]$. Three values of J have been considered: $J = 1, 5, 10$. Each time 4000 iterations of the MCMC algorithm have been performed. The scatter plots of accepted states are depicted in Figure 3. For each session the first ten samples have been discarded in order to allow convergence of the Markov Chain to the stationary distribution ("*burn in*" period). The case $J = 20$ with 12000 iterations is also displayed to illustrate the behaviour of the algorithm for a great number of simulations.

From the figures it can be clearly seen that, for the resolution criterion that has been chosen, there exist two regions of nearly optimal solutions. They correspond to two different encounter geometries, one where aircraft 2 crosses the path of aircraft 1 and one where it does not. Notice that the space where near optimal solutions can be found is separated into two "clouds" and is therefore not convex. Typically this is a problem with optimisation routines which exploit convexity to speed up the search for an optimum solution. It is not an issue with the MCMC algorithm however, which produces an efficient randomised estimate of the optimum. The difficulty is that to ensure that the MCMC algorithm explores all possible near optimal solutions one needs to choose an appropriate search distribution g.

In the remainder of this section we illustrate two resolution maneuvers which belong respectively to the two different regions. For the two maneuvers, conflict probability (P_c) and expected delay between arrivals ($E[\Delta T]$) have been estimated with Monte Carlo simulation by using 10000 trajectory realizations. The first maneuver is determined by

$$\Omega = [-60000 \;\; -40000]$$

for which we estimated $\hat{P}_c = 0$ and $\hat{E}[\Delta T] = 298sec$. The second maneuver is instead determined by

$$\Omega = [38000 \; 60000]$$

and we obtained the estimates $\hat{P}_c = 0.008$ and $\hat{E}[\Delta T] = 304sec$. In Figures 4 and 5 trajectory realizations for the two maneuvers are displayed.

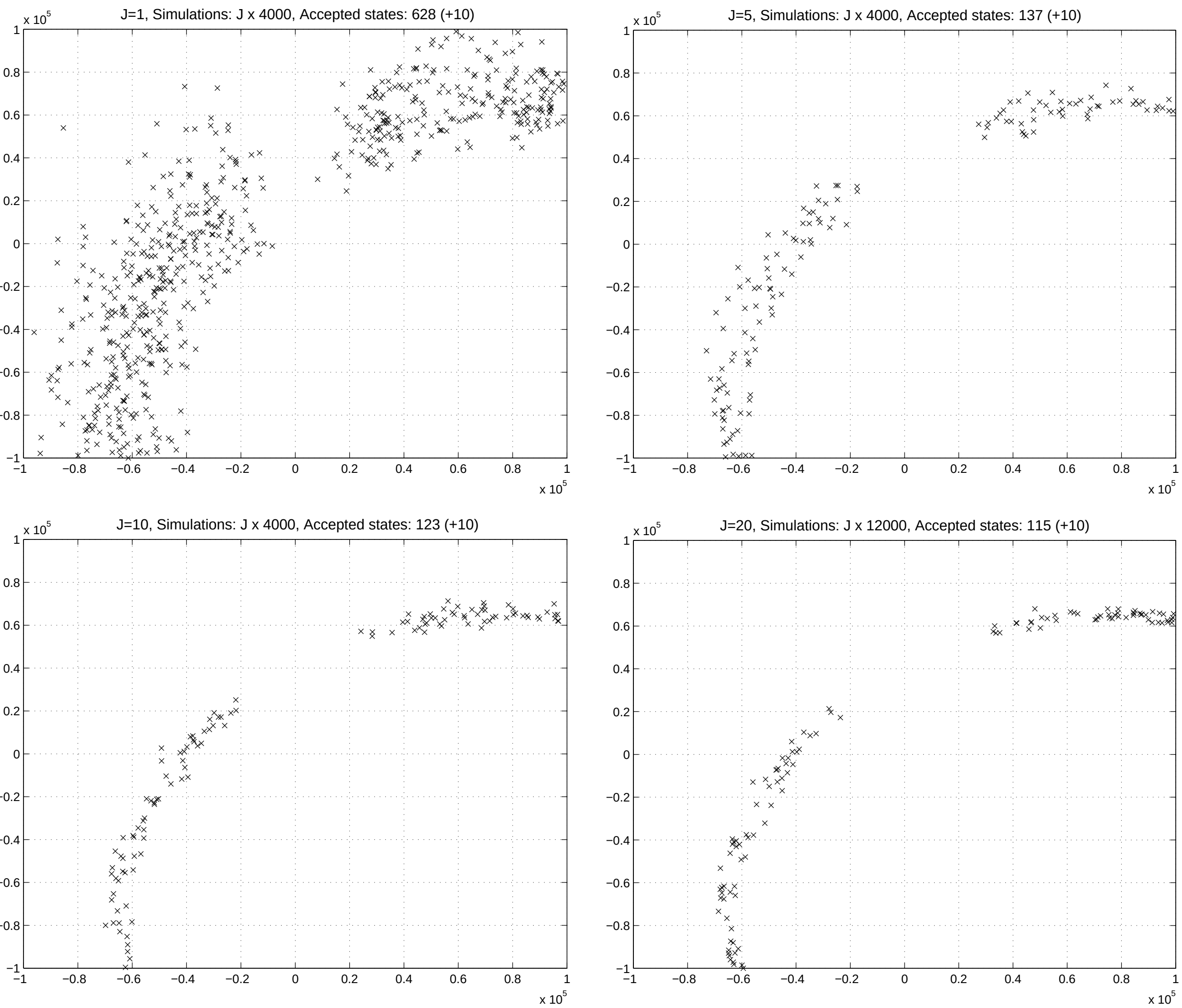

Figure 3: Accepted states during MCMC simulation

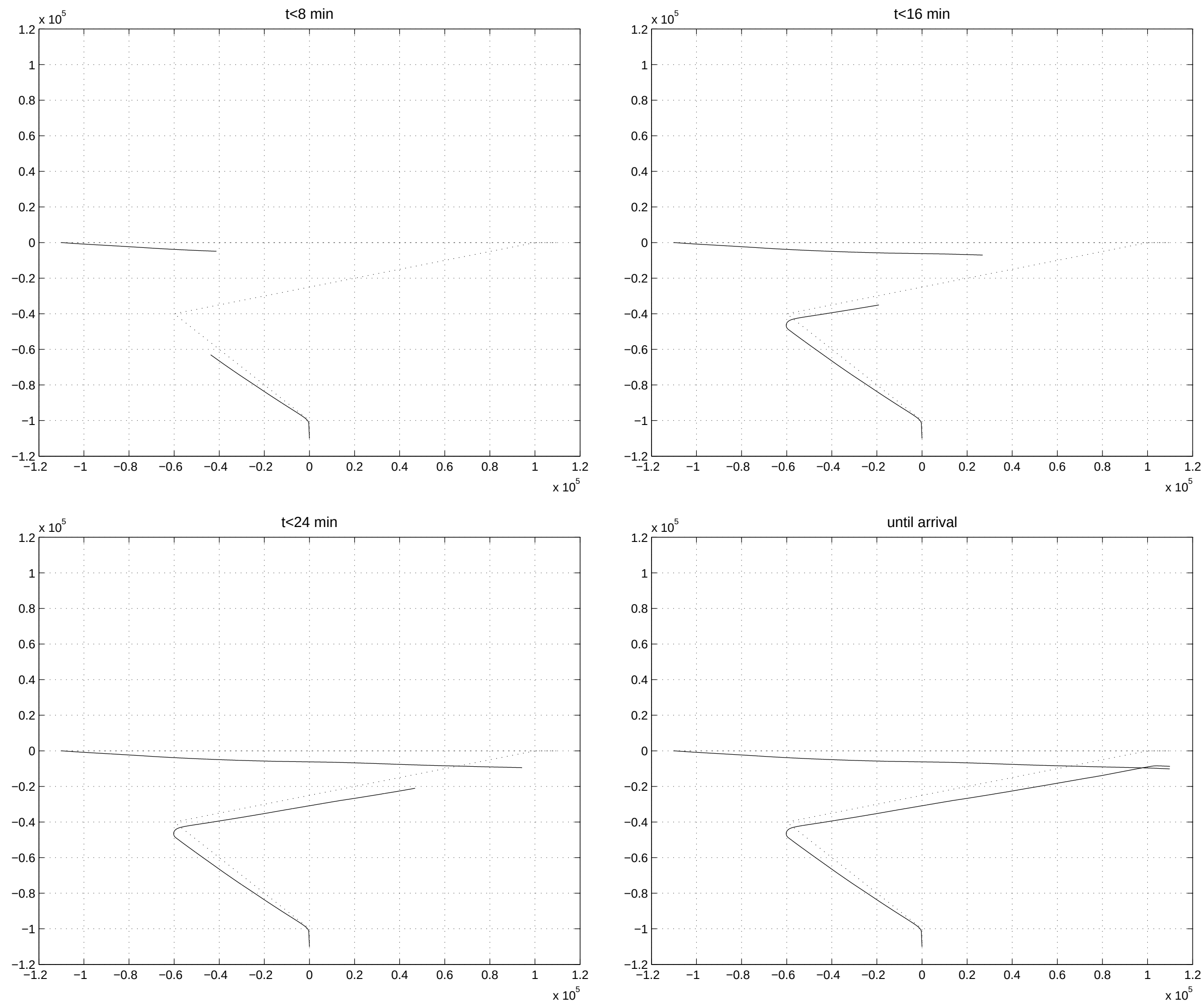

Figure 4: First resolution maneuver: trajectory realizations (continuous) and reference path (dotted)

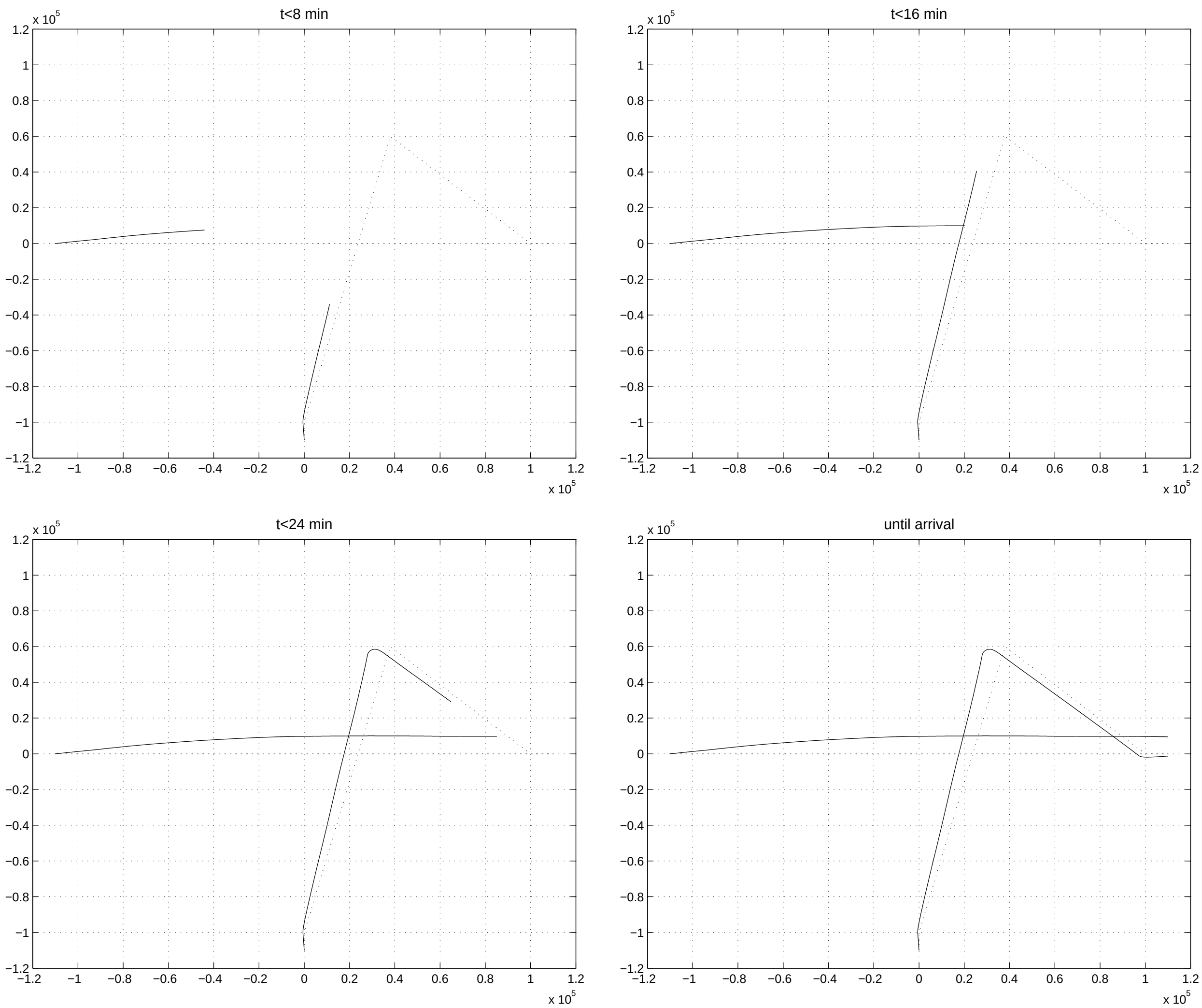

Figure 5: Second resolution maneuver: trajectory realizations (continuous) and reference path (dotted)

7 Conclusions

We discussed a framework for air traffic conflict resolution in a stochastic setting. The resolution of the conflict has been posed as the problem of optimising an expected criterion and a suitable MCMC optimisation procedure to solve the conflict has been described. The MCMC optimisation procedure is simulation based and therefore is quite flexible in the use of prediction models with wind / weather characteristic. The resolution of a simple conflict has been illustrated in a simulation example.

Ongoing research is focused on possible improvements of the resolution algorithm. The speed of the procedure in finding a solution is the main aspect. Obviously there exists a trade off between complexity of the model of the aircraft motion and computation time. Therefore the use of reduced complexity models will be considered. Other aspects under investigation deal with monitoring of convergence of the MCMC to the stationary distribution and possible choices of free parameters in the algorithm in order to increase the convergence speed. These are listed below.

- A proper selection of the resolution criterion $u(\omega, x)$ and of the proposal distribution $g(\omega|\bar{\omega})$. In the simulation example of Section 6 we have used a uniform search distribution. This resulted in time spent to search over regions with a low criterion value and therefore in a great number of rejected samples. In general, the search distribution could include clues on the expected / desired resolution maneuver. This would increase the rate of accepted samples versus rejected samples and the convergence speed of the chain to the stationary distribution [24]. In addition, u could also be used to encode human factor related preferences, such as moving one way point at the time or showing a preference for commands such as vectors that are currently being used by ATC.

- Convergence of the MCMC depends on the initial state of the chain. A part of this state is obtained as radar measurements of aircraft positions. Suitable conditioning with respect to new measurement, whenever new measurements are available, could be included in the algorithm. However, this would require the knowledge of a (simplified) conditional distribution of aircraft position between two successive measurement instants [5, 6, 19, 28].

If the research on the above points will be successful then resolution maneuvers could be selected and monitored and, if necessary, modified during the execution. This would give rise to a complete and efficient Model Predictive Control (MPC) strategy for conflict resolution.

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