HYBRIDGE

Distributed Control and Stochastic Analysis of Hybrid Systems
Supporting Safety Critical Real-Time Systems Design

WP6: Decentralized Conflict Prediction and Resolution

Inventory of Decentralized Conflict Detection and Resolution Systems in Air Traffic

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Inventory of Decentralized Conflict Detection and Resolution Systems in Air Traffic

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Abstract

This is the first deliverable under work package WP6 of the HYBRIDGE project.

This report aims to provide a summary of the most important aircraft Conflict Detection and Resolution (CDR) methods in literature. Motivated by the long-term goals of WP6, which involve decentralized CDR algorithms (and the goals of WP5, 7,8 and 9 that make direct use of the results of WP6), we have focused on distributed CDR methods. Two general categories of such methods have been taken into account: optimization and stochastic-based methodologies, always keeping the distributed part as a priority. Alternative approaches are discussed as well.

In the last part of the report, a brief presentation of the current research efforts taking place in the Control Systems Lab of NTUA is given. These involve the extension of robot navigation methods to the aircraft CDR problem.

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Chapter 1

Introduction and Motivation

1.1 Current ATM Structure-the need for Distributed Methods

Today's air traffic systems remain to a large extent widely centralized [26]. A central authority, namely the Air Traffic Controllers (ATC), is responsible for issuing instructions to conflict-bound aircraft (Fig 1.1). To resolve conflicts they ask aircraft to climb/descend or vector them away from the path in the flight plan and then back on to it. Flight plans are completely pre-defined and aircraft fly along fixed corridors and at specified altitude. Only minor deviations from the original flight plan are permitted on line. Autonomous decision-making by aircraft is allowed under the Traffic Alert Collision Avoidance System (TCAS)[21], which issues advisories in order to avoid potential collisions, yet is used only in extreme situations.

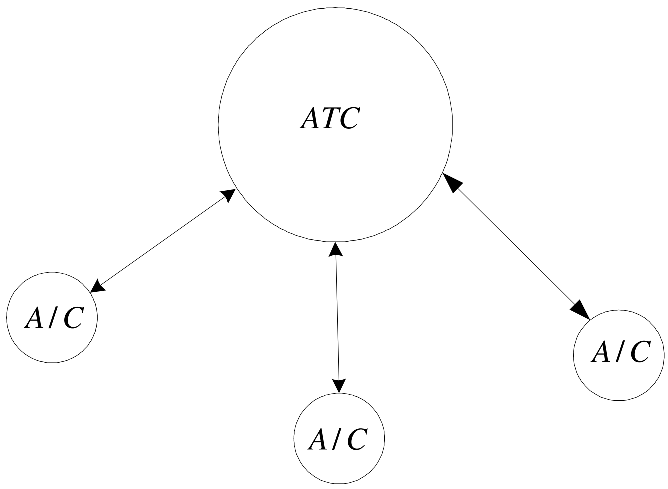

Figure 1.1 Current Centralized ATM Structure

On the other hand, the demand for air transportation is constantly increasing and threatens to exceed the capacity of the current centralized ATM structure. The number of passengers using air traffic is predicted to increase up to 120% in the next ten years [9], and studies in [26] indicate that, with the current ATM structure, a major accident could occur every 7 to 10 days by the year 2015. Moreover, recent technological advances in avionics such as satellite positioning systems (the Global Positioning System (GPS)), inter-communication systems (the Automatic Dependent Surveillance-Broadcast (ADSB)-although its current use in air traffic is rather limited), and powerful on-board computers, are used in terms of the current centralized ATM system and provide an improvement on it, but not a radical change in the air traffic community.

These facts have resulted in the growing will of the air traffic world for new architectures, which employ these new technological innovations towards a more user-centered system. The purpose is to supply pilots with more decisional freedom and to reduce the authority and influence of the ATCs. This has lead air traffic researchers around the globe to a progressively increasing distributed approach to the problem. An excellent comparison between distributed and centralized ATM in terms of system performance is [19].

The power of the ATC is reduced the more autonomy individual aircraft are equipped with, and hence the more distributed the controls governing the system are. Figure 1.2 presents a scheme where exchange of information and coordination between aircraft is allowed. The ultimate purpose of these efforts is *free-flight* [28], a concept in which aircraft will be allowed to plan their en-route trajectories and resolve any conflicts with other aircraft in a distributed and cooperative manner. In this case, the ATC will play the role of a passive observer.

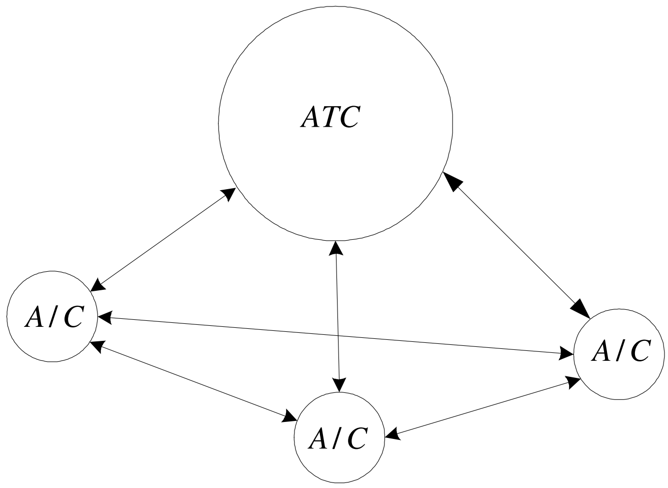

Figure 1.2 A Distributed ATM Structure

A key role in the ATM system is played by *conflict detection and resolution*. We give the following formal definitions:

- A *conflict* occurs whenever two aircraft fly in a distance smaller than a specified separation minimum. The latter is defined at 1000 ft all the way up to 40000 ft on the vertical plane, and at 5 nmi on the horizontal plane.
- A potential conflict is *detected* whenever the trajectories of two aircraft are predicted to lead to a conflict in the future. This requires a mathematical model for the aircraft kinematics in order to make such a prediction.
- The conflict *resolution* procedure involves the actions that should be taken after the detection of a conflict in order to avoid it.

Decentralized conflict detection and resolution involves reassignment of the control tasks from the central authority, i.e. the ATCs, to the agents, i.e. the cockpit. The level of decentralization depends on the knowledge an agent has on the other agents' actions and objectives. In a totally decentralized scheme each agent plans its actions assuming worst-case reaction from its environment. Decrease of decentralization involves some form of cooperation between agents.

The development of decentralized algorithms is therefore crucial for the purposes discussed above. This report aims to summarize the major approaches to decentralized aircraft conflict detection and resolution. The reader is referred to the excellent survey of [20] regarding general CDR methods. Here, we focus on decentralized methods. We distinguish between two major categories: optimization and stochastic based methodologies (Fig1.3). Optimization based methods include non-cooperative game theoretic, cooperative approaches and decentralized optimization approaches. These are discussed in the next chapter. Stochastic methods are discussed in Chapter 3. Alternative methods for aircraft CDR are presented in Chapter 4.

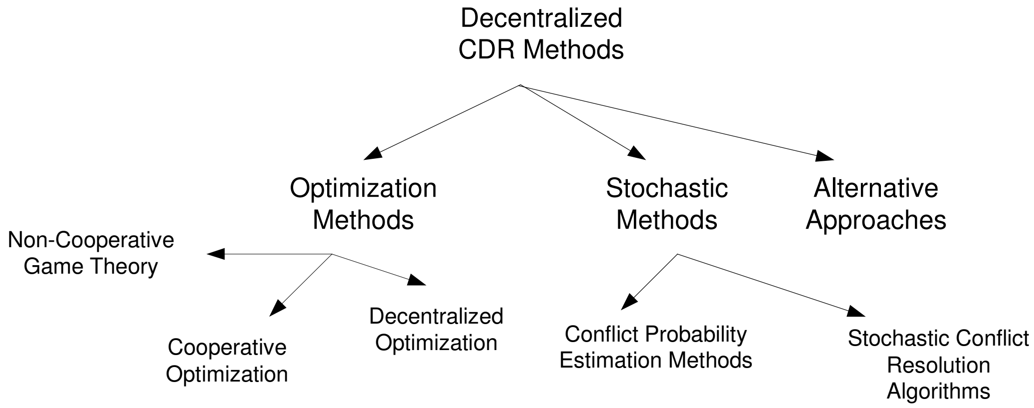

Figure 1.3 Structure of main CDR approaches

1.2 European Projects related to WP6

This section provides a summary of other European projects related to WP6, which serves as a complement to the list of projects in the first deliverable of WP1 [35]. Further details on the following projects can be obtained by http://www.cordis.lu, unless otherwise indicated.

CARE-ASAS- Cooperative Actions of R&D in Eurocontrol. Action on Airborne Separation Assurance Systems. http://www.eurocontrol.int/care/asas/

The action plan involves development of operational procedures towards a progressive transfer of the responsibility for maintaining separation between aircraft from the ATCs to the cockpit. It deals with the operational aspects, not with the supporting technologies (i.e. ADS-B, TIS, TIS-B, air-to-air datalink). It is noted however that the operational scenarios could vary according to the performances of the supporting technologies. Started 1-1-2000.

EMERALD- Emerging Research & Technical Development Activities of Relevance to ATM concept Definition

This project aimed to make recommendations concerning future CNS (Communication, Navigation and Surveillance) systems in a unified European Air Traffic Management System (EATMS). They identified the emerging CNS RTD activities regarding this goal. Completed, 5-11-1998.

MANTEA- Management of Traffic in European Airports

The objective of this project was among others to develop decision support tools for tactical (i.e. near-term) conflict detection and resolution. The adapted algorithms aimed to assist the ATC in carrying tactical decisions in critical situations such as bad weather conditions. Another goal of the project was the optimization of aircraft capacity subject to the constantly growing number of aircraft arriving and departing. Completed, 30-9-1997.

MFF- Mediterranean Free Flight. http://www.medff.it/

The project treats the Mediterranean area as a critical location for the future of air navigation, especially when the concept of Free Flight scenarios are taken into account. The main objectives are the definition, simulation and validation of Free Flight procedures in the Mediterranean area in Free Flight Airspace (FFAS) and between Free Flight Airspace and Managed Airspace (MAS). Started, July 2000. To be completed, December 2004.

NEAP- North European CNS/ATM Application Project

The objectives of this project were the development and evaluation of examples of user applications within the following areas: enhanced surveillance for ATC, pilot situation awareness and GNSS precision navigation capability for all phases of flight. Completed, 31-12-1998.

ONESKY- One Non-National European Sky

The project would present proposals for a European Air Traffic Management system to support efficiency in air traffic regardless of the national frontiers. The proposed ATM would be developed from a "clean sheet" by using a commonly agreed sectorisation logic. Completed, 30-9-2002.

TELSACS- Telematics for Safety Critical Systems

The objectives of this project were the exploitation of new collision avoidance technologies to allow reductions in the standard minimum distances between aircraft to accommodate increased air traffic safely. Cooperative tools would be promoted on-board such as the ACAS (Airborne Collision Avoidance System). Completed, 31-12-1997.

3FMS- Free Flight-Flight Management System

This project aimed to define Free Flight functions to apply to a new European Flight Management System. In the new architecture, pilots will be able to operate the flight safely without specific route, speed, or altitude clearances. New Communication, Navigation and Surveillance (CNS) technologies would be used as tools towards this goal. Completed, 31-5-2002.

Chapter 2

Optimal Control and Game Theory Approaches to CDR

There is a considerable amount of literature in the control community regarding optimization approaches to Aircraft Conflict Detection and Resolution. Different methods from optimal control, non-cooperative game theory and optimization have been proposed in the past few years to deal with the problem of conflict avoidance, each one from a different perspective. In the following paragraphs we make a short review of the most complete methods of that type, focusing on decentralized approaches. The level of decentralization differs in each case. Section 2.1 describes the worst-case approach for two aircraft resolution of [29],[30],[31]. The more cooperative scheme adapted in [4],[5],[6] is presented in section 2.2. Decentralized optimization approaches are discussed in section 2.3. In the analysis that follows in this chapter and all the subsequent chapters, we use the mathematical notation of the corresponding references.

2.1 Non-cooperative Game Theoretic Approach

A natural framework for non-cooperative and thus, distributed, conflict resolution planning is non-cooperative game theory. This approach has been adapted in [29],[30],[31] aiming to resolve problems regarding reachability questions for hybrid systems. In [29] the authors use these results in a case study for conflict resolution of a two aircraft encounter. They calculate the maximal set of initial conditions from which the system state remains within the 'safe' subset of the state space, i.e. the states that guarantee separation between the two aircraft, in the presence of worst-case uncertainty in the actions of the other aircraft.

The relative position and heading $x = (x_r, y_r, \psi_r) \in \mathbf{R}^2 \times [-\pi, \pi)$ of aircraft 2 with respect to aircraft 1 is given by the following kinematic model:

$$\begin{aligned} \dot{x}_r &= -v_1 + v_2 \cos\psi_r + \omega_1 y_r \\ \dot{y}_r &= v_2 \sin\psi_r - \omega_1 x_r \\ \dot{\psi}_r &= \omega_2 - \omega_1 \end{aligned} \qquad (2.1)$$

where v_i is the linear velocity of aircraft i and ω_i its angular velocity (Fig. 2.1). The safe subset of the state space is defined as:

$$F = \{(x_r, y_r, \psi_r) : l(x) = x_r^2 + y_r^2 - 5^2 \geq 0\} \qquad (2.2)$$

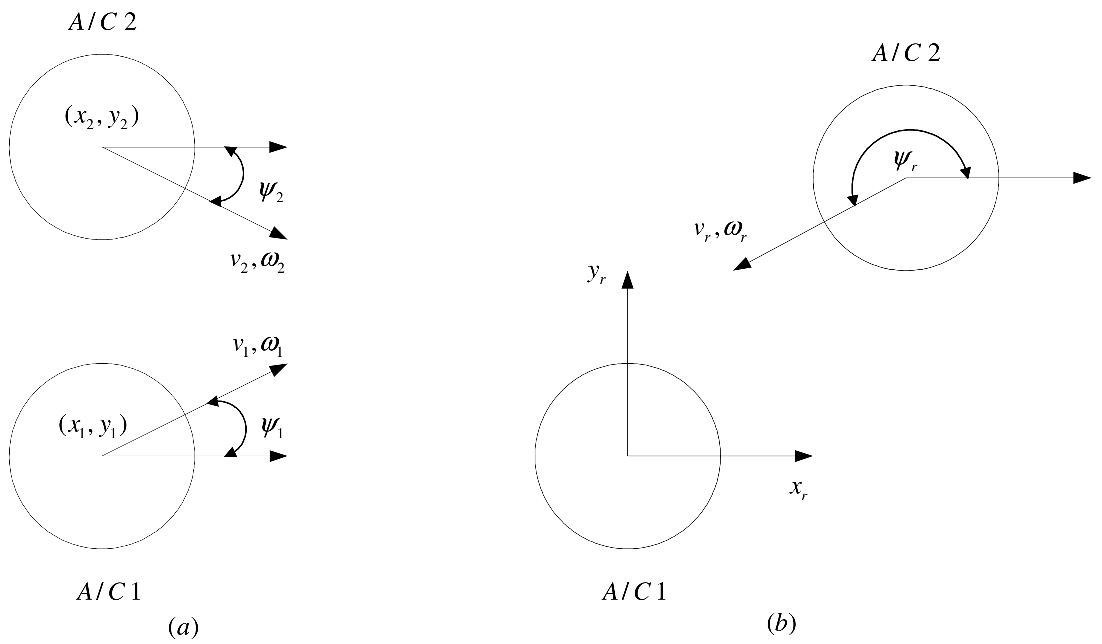

Figure 2.1 (a) A conflict encounter between two aircraft. (b) The relative movement of aircraft 2 with respect to aircraft 1.

Following the hybrid system model definition the actions of aircraft 1 are considered as the control input $u \in U$ and the uncertain actions of aircraft 2 the disturbance $d \in D$. In [31], for the case in which the linear velocities of both aircraft are fixed, the control input and disturbance are the angular velocity of aircraft 1, respectively aircraft 2, i.e. $u = \omega_1, d = \omega_2$. In that case, the sets U and D are simply defined within the bounds of the angular velocities, i.e. $U = [\omega_{1\min}, \omega_{1\max}], D = [\omega_{2\min}, \omega_{2\max}]$.

The goal of the controller is to keep the system state outside of the 'unsafe, set F^C throughout the encounter, regardless of the disturbance actions. The conflict resolution scheme in [29] is modeled as a hybrid system consisting of three modes (discrete states) of operation: a cruising mode before the avoidance maneuver, a conflict avoidance mode, and a cruising mode after the completion of the avoidance maneuver (Fig. 2.2).

Reachability analysis for hybrid systems is used in [29],[30] to compute the unsafe subset of the state space from which there exists a disturbance action d such that the trajectory enters $F^C \cup T$. The algorithm of [30] regarding only one discrete state is extended in [29] to general nonlinear hybrid systems. The framework for the analysis in the continuous domain is two agent-zero sum differential game theory. The cost of the game in the time interval $[t,0]$, $t<0$ is defined as the value of $l(x)$ at 0:

$$J(x,u(\cdot),d(\cdot),t): \mathbf{R}^n \times U \times D \times \mathbf{R}_- \to \mathbf{R}, \text{ such that } \quad J(x,u(\cdot),d(\cdot),t) = l(x(0)) \qquad (2.3)$$

The controller (aircraft 1) wins the game if it maintains the minimum allowable separation from aircraft 2 throughout the encounter. Thus, the controller tries to maximize the worst actions of the disturbance (aircraft 2), and the disturbance to minimize the optimal actions of the controller:

$$u^* = \arg \max_{u \in U} \min_{d \in D} J(x, u(\cdot), d(\cdot), t)$$
$$d^* = \arg \min_{d \in D} \max_{u \in U} J(x, u(\cdot), d(\cdot), t) \tag{2.4}$$

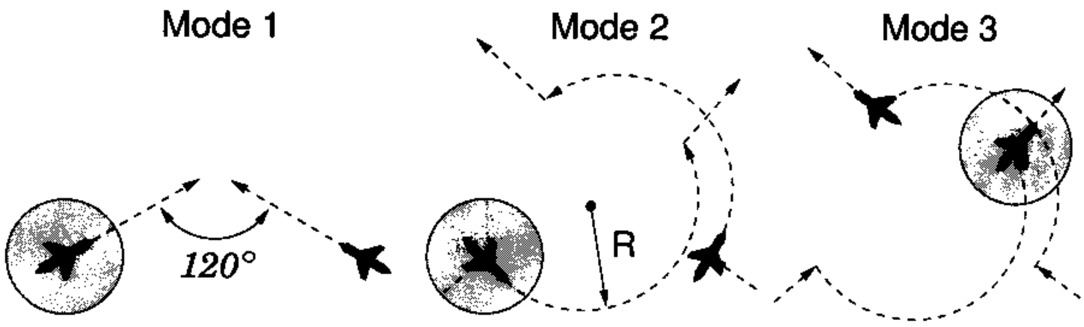

Figure 2.2 Two Aircraft Conflict Resolution Process
in three modes of operation. [31 ,Fig. 2.8, p.23]

The game admits a 'saddle solution' in feedback strategies if the resulting cost does not depend on the order the maximization and minimization are performed:

$$J^*(x,t) = \max_{u \in U} \min_{d \in D} J(x, u(\cdot), d(\cdot), t) = \min_{d \in D} \max_{u \in U} J(x, u(\cdot), d(\cdot), t) \tag{2.5}$$

If J^* is a smooth function of x,t then it satisfies the Hamilton-Jacobi-Isaacs equation:

$$-\frac{\partial J^*(x,t)}{\partial t} = H^*(x, \frac{\partial J^*(x,t)}{\partial x}) \tag{2.6}$$

where $H^*(x,p) = \max_{u \in U} \min_{d \in D} p^T f(x,u,d)$ the optimal Hamiltonian of the game and $f(x,u,d)$ the system dynamics (2.1). The unsafe subset of the state space is then characterized as

$$Pre_{-\infty}(T) = \{x \in Inv(q) \mid J^*(x,\tau) < 0, \forall \tau \in (-\infty, 0)\} \tag{2.7}$$

where $Inv(q)$ the invariant of the discrete state q with respect to the hybrid systems literature. In order to ensure that states, which are once unsafe, cannot become safe $Pre_{-\infty}(T)$ is calculated by solving a modified version of (2.6):

$$-\frac{\partial J^*(x,t)}{\partial t} = \min\{0, H^*(x, \frac{\partial J^*(x,t)}{\partial x})\} \tag{2.8}$$

In the interior of $\{Pre_{-\infty}(T)\}^C$ any control input can be applied by aircraft 1 to maintain separation, whereas on the boundary, the control input must force the system outside the unsafe set, regardless of the disturbance actions of the other aircraft. The least restrictive feedback controller [29] is then given by:

$$u \in \begin{cases} U, if\ x \in \{Pre_{-\infty}(T)\}^C \\ \{u \in U \mid \min_{d \in D} \frac{\partial J^*(x,t)}{\partial x} f(x,u,d) \geq 0\}, if\ x \in \partial Pre_{-\infty}(T) \end{cases} \quad (2.9)$$

In [30], two constraints on control inputs are then considered. The first considers changes only on the linear velocities and the second one only on the angular velocities. Under these assumptions the equation (2.8) can be solved analytically and thus, the sets $Pre_{-\infty}(T)$, $\partial Pre_{-\infty}(T)$ and the feedback law (2.9) can be calculated.

An extension of this approach to the case in which the manifold over which the aircraft dynamics evolve is a Lie group has been presented in [32]. Safety verification of this method for conflict resolution is discussed in [33].

Comments: The purpose of this approach is not on line conflict resolution, but prediction of flight plans in which conflict is impossible. While for the general case the algorithm does not guarantee to converge, for the simple dynamics of the two aircraft encounter solutions have been computed analytically. It is highly unlikely that this method would produce useful results for *N*>2 aircraft. A similar discussion of this method was also given in HYBRIDGE D1.1 [35].

2.2 Cooperative Optimization Approach based on Optimal Robotic Path Planning

In contrast to the total uncooperative scheme of the previous section, in what follows we present a partially decentralized optimization based framework developed in [4],[5],[6]. The authors consider the problem of steering *N* aircraft among 2*N* given waypoints. The planar motion of the *i*-th aircraft is characterized by the nonholonomic model:

$$\begin{aligned} \dot{x}_i &= u_i \cos\theta_i \\ \dot{y}_i &= u_i \sin\theta_i \\ \dot{\theta}_i &= \omega_i \end{aligned} \quad (2.10)$$

where $\xi_i = (x_i, y_i, \theta_i)$ is the state vector, u_i is the linear velocity, regarded constant throughout the encounter for each aircraft, and $\omega_i \in [-\Omega_i, \Omega_i]$ the angular velocity. Let ξ_s^i, ξ_g^i denote the start and goal waypoints for aircraft *i*. Assuming that all aircraft are at their starting waypoints at the same time, the time at which the *i*-th aircraft reaches its goal is denoted by T_i. A collision occurs whenever the “safety discs” of at least two aircraft intersect. The safety disc of aircraft *i* has radius D_i and is centered at the position of the aircraft.

In the proposed decentralized setup the following assumptions are made:

- the i-th aircraft has information on the state and goals of all other agents which are at a distance smaller than an "alert" radius $R_i > D_i$ (Fig.2.3) ;
- each aircraft plans its path aiming to minimize the sum of the time-to-goals of all neighboring aircraft.

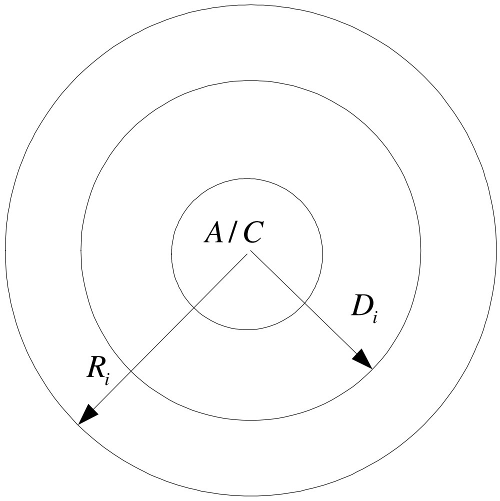

Figure 2.3 Safety and Alert Discs around an aircraft

Let $S_i(\tau)$ denote the indices of aircraft within the alert zone of aircraft i at time τ :

$$S_i(\tau) = \{ j \mid C_{ij}(\xi_i, \xi_j) = (x_i - x_j)^2 + (y_i - y_j)^2 - R_i^2 \le 0\} \qquad (2.11)$$

Therefore at time τ, the i-th aircraft plans its actions according to the following optimization problem:

$$\min_{\omega_j \mid j \in S_i} J_{i,S_i}(\tau), \text{ where } J_{i,S_i}(\tau) = \sum_{j \in S_i} \int_{\tau}^{T_i} dt \qquad (2.12)$$

s.t. the constraints

$$\begin{aligned} V_{ij}(\xi_i, \xi_j) &= (x_i - x_j)^2 + (y_i - y_j)^2 - D_i^2 > 0, \\ \omega_j &\in [-\Omega_j, \Omega_j] \end{aligned} \quad \forall j \in S_i \qquad (2.13)$$

The level of decentralization of the problem depends on the size of the alert radius R_i. The bigger the considered radius, the more centralized the problem becomes. At each time τ, the system admits a certain information structure $I = (S_1, ..., S_N)$. Whenever an aircraft $i \notin S_j$ enters in the alert zone of j, the information structure and the optimal paths are updated in real time by all agents. Therefore to each different information structure I_k there corresponds a different working mode of the system, i.e. the system can be modeled as a hybrid automaton with continuous variables

$\xi_i, \omega_i, i = 1,..., N$ and discrete variables S_i (Fig. 2.4). Transitions between the discrete modes I_k are triggered by the discrete events described above. At every state transition, each agent re-plans its optimal steering control from its current position to the desired destination as well as for all other aircraft within its alert radius.

An algorithm is proposed in [4], [5] to solve the optimal control problem described above, firstly in the centralized setting ($R_i \to \infty$). The algorithm is based on the well-known from robot motion planning literature "Dubins' car", i.e. a vehicle that can only go forward and has bounded curvature. The shortest path between two way points for such a vehicle is called a "Dubins' path". The authors propose a suboptimal solution which consists of a concatenation of unconstrained Dubins' paths for each aircraft, i.e. paths in which $V_{ij} > 0, \forall i, j$ and at most one zero length constrained path, i.e. a path in which $\exists (i, j) : V_{ij} = 0$. Further details are omitted here. The reader is referred to [4] for an analytic description.

This approach is used then in the decentralized setup as well and a comparison of the two methods is made. Simulation results for varying alert radii R_i show that while the decentralized procedure involves bigger fuel consumption, it is more robust with respect to fault tolerance than the centralized scheme.

Comments: This methodology seems more realistic than the totally non-cooperative approach. Fuel consumption and economy matters are taken into account. However, the solution scheme is suboptimal and the complexity grows combinatorially as the number of aircraft increases.

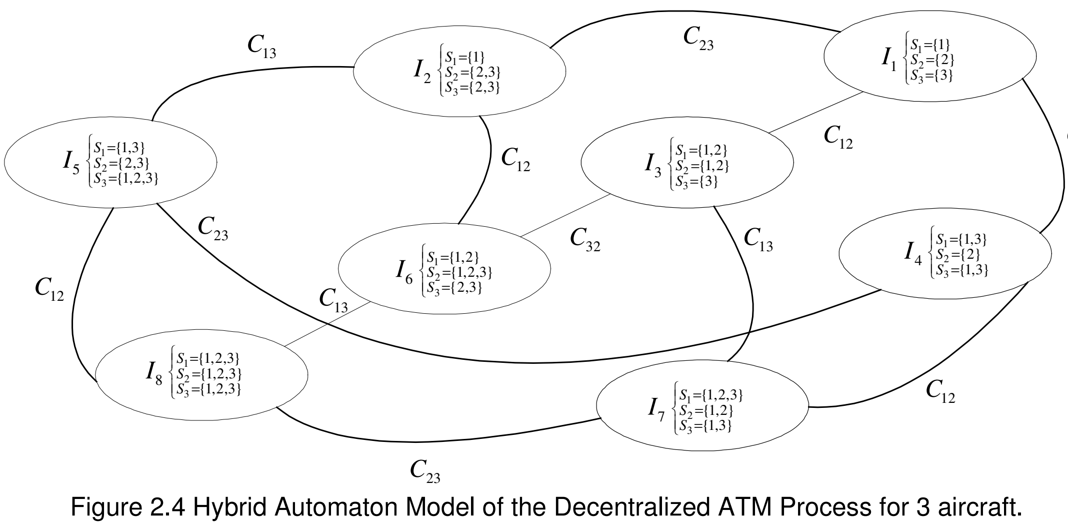

Figure 2.4 Hybrid Automaton Model of the Decentralized ATM Process for 3 aircraft. [5, Fig. 2]

2.3 Decentralized Optimization Approaches for Conflict Detection and Resolution

An important part of recent work on optimization based methods for conflict resolution doesn't take into account the decentralized nature of the problem ([10], [12], [14]). In fact, there is a whole area of centralized CDR where optimization has been applied, namely in Ground Holding methods for Air Traffic Flow Management (ATFM). The idea is simple: whenever the arrival capacity at destination airports is low, it is preferable to have a flight wait its origin than to have it circle the destination airport, unable to land. The optimal planning is achieved by assigning ground delays to incoming aircraft so that the arrival flow will match the forecasted capacity. This topic will not be discussed further in this report, since we mainly focus on decentralized methods. For further discussion, the reader is referred to [2] and [3].

Decentralized optimization approaches include [1], [15] and [23]. In [1], the authors propose a priority-based optimization algorithm, in which the first agent chooses its trajectory without considering the others, the second takes this trajectory into account and computes its own, and so on. The underlying scheme is formulated as a problem of finding the shortest path in a tree. In what follows we describe briefly the work in [15] and [23], in which the authors treat the problem in a distributed manner.

A similar approach to that of section 2.2 to aircraft conflict resolution is developed in [23] but its solution is not carried out in the same way. Here the authors consider 4-dimensional aircraft trajectories, time being the fourth dimension, and use the Brunovsky's canonical form to describe the kinematics of each aircraft:

$$\ddot{x} = U_1, \ddot{y} = U_2, \ddot{h} = U_3 \tag{2.14}$$

where (x, y, h) the position of the center of gravity of the aircraft with respect to a ground reference frame and U_i the control inputs of the point-mass model. The safety disc of the planar case is replaced by an oblate spheroidal conflict envelope. A conflict occurs whenever the distance of an aircraft i 's conflict envelope and another aircraft j's center of gravity falls below zero, i.e. when $r_{ij} < 0$, where

$$r_{ij} = \sqrt{\Delta x_{ij}^2 + \Delta y_{ij}^2 + \Delta h_{ij}^2} - \sqrt{\frac{a^2 b^2 (\Delta x_{ij}^2 + \Delta y_{ij}^2 + \Delta h_{ij}^2)}{a^2 \Delta h_{ij}^2 + b^2 (\Delta x_{ij}^2 + \Delta y_{ij}^2)}} \tag{2.15}$$

where a is the conflict envelope semimajor axis, b the semiminor axis, and $\Delta x_{ij}, \Delta y_{ij}, \Delta h_{ij}$ the components of the relative position vector between the two aircraft. The authors distinguish between two approaches to the conflict resolution problem: a single-objective (centralized) and a multi-objective (decentralized) optimization formulation.

In the first approach, starting from a predefined sequence of nominal waypoints, all aircraft cooperate in order to resolve conflicts while minimizing a performance index. This is the sum of integral deviations of all aircraft from their nominal trajectories.

The usual inequality constraints are used and the resulting problem is in the same vein with the centralized version of the formulation discussed previously. The solution here is computed using the Sequential Quadratic Programming (SQP) method. This method deals with optimization problems of the form

$$\begin{aligned} &\min f(x) \\ &s.t.\ g(x) \leq 0 \end{aligned} \tag{2.16}$$

The cost function $f(x)$ is approximated by a quadratic function and the inequality constraints $g(x)$ by linear functions.

In the second approach each aircraft (or a team of aircraft) has its own objectives and therefore the single objective optimization approach is not suitable in this case. The cost function is now a vector and reflects the multi-objective nature of the problem. The problem is solved using the goal attainment method in which constrains and objectives are represented as goals to be satisfied. This method is stated as:

$$\begin{aligned} &\min \gamma, \\ &where\ f(x) - w\gamma \leq f^{*} \end{aligned} \tag{2.17}$$

where $f(x)$ is the vector that includes the performance indices and the constraints, f^{*} represents the goals, w is a weight vector, and γ is a measure of how far a solution is from the goals. The goal attainment method can be treated as an SQP problem.

Simulation results in [23] show that the multi-objective (decentralized) approach results in more robust solutions in this case as well.

A decomposition approach has been recently developed in [15] and treats the problem of decentralized optimization of a system with local dynamics and global constraints. The results are then applied to the multiple aircraft conflict resolution problem. The authors first consider a discretized nonlinear system of the form

$$z(k+1) - H_d(z(k), u(k)) = 0,\ k = 0,1,...,l-1 \tag{2.18}$$

consisting of the subsystems

$$z_i(k+1) - H_{di}(z_i(k), u_i(k)) = 0,\ k = 0,1,...,l-1,\ i = 1,...,m \tag{2.19}$$

The discrete time local state vector is defined as

$$x_i = [x_i^T(0),...,x_i^T(l-1)]^T,\ i = 1,...,m \tag{2.20}$$

where $x_i(k) = [z_i^T(k), u_i^T(k)]^T$. The global state vector is defined as

$$x = [x_1^T,...,x_m^T]^T \tag{2.21}$$

The difference equations (2.19) can be combined with equality and inequality constraints on states and inputs of the system to give the following constraints for the optimization process:

$$h(x) = 0, g(x) \le 0 \tag{2.22}$$

The neighborhood of the *i*-th subsystem is defined as $N_i = \{ j : i^{th}$ subsystem has a global constraint involving the j^{th} subsystem$\}$. The authors use the following notation:

$$\{x_j\}_i = \{\tilde{x}_j \subset x_j \mid j \in N_i\} \tag{2.23}$$

which is the set of subsets of neighborhood states that the *i*-th subsystem is associated with. The decentralized optimization problem is then stated as:

$$\min_{x_i} f_i(x_i)$$
$$s.t. \begin{cases} g_i(x_i \mid \{x_j\}_i) \le 0 \\ h_i(x_i \mid \{x_j\}_i) = 0 \end{cases} \tag{2.24}$$

where $g_i(x_i \mid \{x_j\}_i), h_i(x_i \mid \{x_j\}_i)$ represent constraints on x_i given that the neighborhood states are fixed.

The authors present then an algorithm based on Lagrange multipliers and penalty function methods. They prove that their algorithm terminates in a finite number of iterations and converges globally. Furthermore they provide conditions under which the derived solution is of the Nash equilibrium type. An application to a 4-aircraft encounter is then made and simulation results are given.

Comments: The latter is the most complete work of that type. Numerical issues are again the drawback of such methods, however, the authors in [15] treat this in a satisfying manner. What is most promising for this method, is its direct applicability to *N*>2 aircraft encounters.

2.4 General Conclusions on Optimization Methods for CDR

The methods described in the previous sections provide a summary for optimization-based approaches to CDR. General conclusions can be drawn after reviewing these works:

- Optimization techniques are the natural framework to deal with CDR, when matters such as fuel consumption and passenger comfort are taken into account.
- Decentralized optimization methods yield more robust solutions than centralized approaches.
- The numerical implementation of the algorithms is as usual not trivial, a fact that is common in optimization problems with state and input constraints.

- The computational complexity of the algorithms grows combinatorially as the number of aircraft increases even in decentralized algorithms, where an aircraft takes into account only the intruding aircraft in its "neighborhood".

Chapter 3

Stochastic Models for CDR

In the following we make a brief overview of the most efficient approaches to probabilistic CDR. A major contributor to the uncertainty in ATM is the wind, for which a complete mathematical model is not available yet. However, in most cases the uncertainty factor is just a Gaussian noise process, which is added to the nominal (i.e. deterministically predicted) position of the aircraft. Hence, the distribution of the prediction error is assumed to be normal. This assumption was verified in [25] using actual air traffic data. The major goal of most of these methods is to compute the Probability of Conflict (PC), in a two aircraft encounter in general. Other performance metrics are considered in some cases as well (e.g. [8]) but PC is the most common one. In section 3.1 some well-known methods of PC estimation are discussed. In section 3.2 we make a short description of the stochasticity-based algorithm for CDR of [27].

3.1 Conflict Probability Estimation Methods

The estimation of Conflict Probability is of major importance in ATM research mainly due to two reasons: it can be used as a threshold for a conflict prediction alert and therefore, for the initiation of a conflict resolution procedure, and secondly, it can be considered as a performance metric for the validation and comparison of the various CDR methods in literature. Various methods have been proposed in the past few years for the estimation of this metric. We make a brief discussion on these methods.

A well-established model for PC estimation in a two aircraft encounter is due to Paielli and Erzberger [24]. An estimation of PC is required for the determination of the optimal time to initiate a maneuver in a conflict resolution scenario, and the optimal choice of the type of this maneuver. The authors make the following important assumptions:

- the prediction errors are approximated as normally distributed
- the planned velocities and prediction errors of both aircraft are considered constant throughout the encounter.

The first assumption allows the combination of the two error covariances of the aircraft pair into a single covariance of their relative position. The combined covariance is assigned to one of the aircraft, the ‘stochastic’ aircraft *S*, while the other aircraft (‘reference’ aircraft *R*) is regarded not to have position uncertainty. The prediction error of the position difference is:

$$\Delta \vec{p} = \vec{p}_S - \vec{p}_R \tag{3.1}$$

where $\vec{p}_S, \vec{p}_R$ the position prediction errors of each aircraft. The combined error covariance is then

$$M \equiv \mathrm{cov}(\Delta \vec{p}) = Q_S + Q_R - Q_{SR} \qquad (3.2)$$

where Q_S, Q_R the individual covariances of the prediction errors of each aircraft and the cross-correlation term is given by $Q_{SR} = E(\vec{p}_S \vec{p}_R^{\,T} + \vec{p}_R \vec{p}_S^{\,T})$.

The combined error covariance can be represented as an ellipse centered on the stochastic aircraft whereas the circular conflict zone (5 nmi radius) is centered on the reference aircraft. The probability of conflict at a particular time is the portion corresponding to the intersection of these two surfaces. What is of main interest is the *total probability* of the encounter which is the portion corresponding to the intersection of the combined error ellipse and the extended conflict zone. The latter is formed by projecting the circular conflict zone along a line parallel to the relative velocity of the two aircraft.

A coordinate transformation is then used in such a way that the combined error ellipse transforms into a unit cycle. This is achieved due to the fact that in the transformed system the combined error covariance matrix is the unity matrix. The circular conflict zone becomes an ellipse in the new coordinates. This leads to an analytical computation of the total PC. The transformation is held in such a way, that the relative velocity is in the positive (or negative) x-direction. The boundaries of the extended conflict zone are then the minimum and maximum values of y (denoted $\pm\Delta y_c$) on the elliptical conflict boundary and are computed analytically.

Because of the fact that the combined error covariance admits the form of a unit circle the corresponding 2-D density function decouples into the product of two identical 1-D density functions: $p(x, y) = p(x)p(y)$, where $p(x) = \exp(-x^2/2)/\sqrt{2\pi}$. The total PC is then given by the following expression (for further details see [24]):

$$P_c = \int_{-\Delta y-\Delta y_c}^{-\Delta y+\Delta y_c} p(y)dy = P(-\Delta y + \Delta y_c) - P(-\Delta y - \Delta y_c) \qquad (3.3)$$

where Δy is the relative y-coordinate of S with respect to R, and P is the cumulative normal probability function.

The probability of conflict derived in closed-form in (3.3) can then be used a criterion for the optimal time to initiate a maneuver. This according to the authors can be determined by minimizing a cost function, which depends on this conflict probability as well as other issues such as operational cost and passenger comfort. A conflict is resolved by moving the extended conflict zone sufficiently far away from the center of the error circle in order to reduce the conflict probability to some desired level.

An extension of this approach to 3-dimensional flight is found in [25].

A similar approach to the problem can be found in [16]. In a planar conflict encounter the aircraft X is centered at the origin of a transformed coordinate system, heading in the positive x-axis and aircraft Y is flying at relative heading angle θ between the two. The encounter geometry is shown in figure 3.1. Here O denotes the crossing point, x (resp. y) the along-track distance flown by X (resp.Y) at time t, and P_x (resp. P_y) the position of X (resp.Y). The two aircraft will be in conflict if Y lies within a circle of radius s=5 nmi centered on the position of X. Application of the cosine rule to the triangle OP_xP_y yields:

$$x^2 + y^2 - 2xy\cos\theta = s^2 \quad (3.4)$$

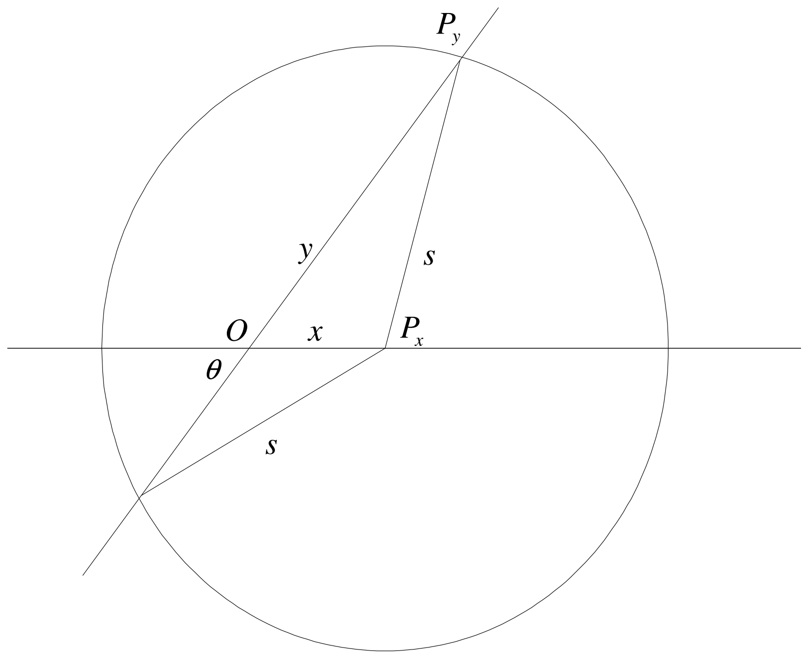

Figure 3.1:Encounter Geometry in the Planar Case [16, Fig. 1]

Equation (3.4) represents an ellipse in the plane. The locus of the along-track distances (x,y) of the two aircraft defines a path in the diagram of the ellipse. The separation minimum s is held throughout the encounter if this path passes outside the ellipse and vice versa. Assuming constant velocities for each aircraft in the conflict alert interval this path is a straight line whose slope $m = v_{PY} / v_{PX}$ is the ratio of the speeds of the two aircraft. A conflict will occur if the straight-line path lies between the two tangents to the ellipse. Simple analytical geometry is then used to determine the condition under which a given point ($x^{'}, y^{'}$) is between the two tangents:

$$-c < y^{'} - mx^{'} < c, c = s\frac{\sqrt{m^2 - 2m\cos\theta + 1}}{\sin\theta} \quad (3.5)$$

There are two cases of singularities, namely $\theta = 0$ and $\theta = \pi$. In the first case, conflict avoidance is guaranteed provide that the relative position of the two aircraft is bigger than s, while in the second case, conflict is inevitable [16].

The uncertainty is then considered firstly in the along-track error. An analogous analysis yields a closed-form expression for the probability of conflict due to the cross-track error as well. In a similar manner with [24], the along-track prediction error is assumed to be approximately constant in an aircraft's position and the aircraft are assumed to fly with their predicted constant speed. Let $a_X(\tau), a_Y(\tau)$ denote the predicted position error of aircraft X,Y respectively at time τ. Under the above assumptions these errors can be considered as errors in the initial along track distances:

$$
\begin{aligned}
x' &= x_0 + a_X(\tau) \\
y' &= y_0 + a_Y(\tau)
\end{aligned}
\tag{3.6}
$$

The predicted position errors $a_X(\tau), a_Y(\tau)$ are assumed to be normally distributed:

$$
a_j(\tau) = a\tau A_{j,\tau} \tag{3.7}
$$

where $A_{j,\tau}$ a normally distributed random variable and a the rate of growth of the standard deviation of the along track error. Following the same procedure as for the nominal (i.e. deterministic) case discussed previously the probability of conflict is derived in closed-form as:

$$
P_C = \frac{1}{\sqrt{2\pi}} \int_{(-c-\mu_i)/\sigma_i}^{(c-\mu_i)/\sigma_i} \exp(-z^2/2)dz = P((c-\mu_i)/\sigma_i) - P((-c-\mu_i)/\sigma_i) \tag{3.8}
$$

where $\mu_i = y_0 - mx_0, \sigma_i^2 = a^2\tau^2(1+m^2)$ the mean and variance of the y-intersect (which is now a random variable) of the line with gradient m passing through (x', y'), and P cumulative normal probability function. The reader can notice the similarity of equations (3.3), (3.8).

An equivalent approach is found in [18] where the authors conclude to a similar type of closed-form expression for the Probability of Conflict in a two aircraft encounter. Here PC is related with conflict resolution maneuvers, which are proposed at a *strategic* (i.e. long term) level and the criteria of choice are the economics of the maneuver, while safety is maintained as a constraint. Three types of maneuvers are considered: heading, speed and altitude change maneuvers. Simulation results show that altitude change maneuvers are the less expensive, while speed change maneuvers are the most costly.

An interesting method is described in [8] where the following stochastic model for a N-aircraft motion is adapted:

$$
dx_t^i = f^i(x_t, \theta_t, t)dt + g^i(x_t, \theta_t, t)dw_t^i \tag{3.9}
$$

where $x^i \in \mathbf{R}^n$, θ_t a finite state process such that $\{x_t, \theta_t\}$ is Markov process and $\{w_t^i\}$ a standard n-dimensional Brownian motion. The 3-D motion of aircraft i is formed by $s_t^i = Hx_t^i$, where H a $3\times n$ matrix. Assuming that $Hg^i(x_t, \theta_t, t) = 0, \forall i$ we have $ds_t^i = v_t^i dt, v_t^i = Hf^i(x_t, \theta_t, t)$. The relative position of two aircraft i and j is a process $s_t = s_t^i - s_t^j$ described by the following stochastic differential equation:

$$ds_t = v_t dt,\ v_t = v_t^i - v_t^j \tag{3.10}$$

The instantaneous conflict probability $P_{ic}(t)$ at time t is defined as the probability that two aircraft that at time *t* fly in the same altitude ($s_{\perp,t} = 0$) will be in a distance less than the conflict radius *d*. Let $S = diag\{1,1,a_\perp\}$, where $a_\perp$ a vertical position scaling factor. Then,

$$P_{ic}(t) \equiv \Pr\{\|Ss_t\| \le d\} = \iint_{\|Ss_t\| \le d} p_{s_t}(s)ds \tag{3.11}$$

where $p_{s_t}(s)$ the predicted probability density function of the relative position s_t at time *t*. Assuming that the predicted relative horizontal position $s_t^{'}$ at time *t* is Gaussian with mean $\mu_s^{'}(t)$ and positive definite covariance $Q_s^{'}(t)$, that the predicted relative vertical position at time t $s_{\perp,t} = 0$ then $P_{ic}(t)$ is given by:

$$\begin{aligned} P_{ic}(t) = \int_{\bar{y}_2(t)-d}^{\bar{y}_2(t)+d} [\frac{1}{2}\{Erf(\frac{\bar{y}_1(t) + g(y_2, \bar{y}_2(t), d)}{\sqrt{2\lambda_1(t)}}) - \\ -Erf(\frac{\bar{y}_1(t) - g(y_2, \bar{y}_2(t), d)}{\sqrt{2\lambda_1(t)}})\} \cdot \frac{\exp[-y_2^2 / 2\lambda_2(t)]}{\sqrt{2\pi\lambda_2(t)}}]dy_2 \end{aligned} \tag{3.12}$$

where

$$g(y_2, \bar{y}_2, d) = \sqrt{d^2 - (y_2 - \bar{y}_2)^2},$$
$$[\bar{y}_1(t), \bar{y}_2(t)]^T = -R(t)\mu_s^{'}(t),$$
$$diag(\lambda_1(t), \lambda_2(t)) = R(t)Q_s^{'}(t)R^T(t)$$

and $R(t)$ a 2×2 rotation matrix such that $R(t)Q_s^{'}(t)R^T(t)$ is a diagonal matrix.

Erf(*x*) is of course the error function $Erf(x) \equiv \frac{2}{\sqrt{\pi}}\int_0^x e^{-z^2}dz$.

The probability of conflict in a time interval $[t_0, t_1]$ is given by:

$$P_c(t_0, t_1) \equiv \max_{t\in[t_0,t_1]} P_{ic}(t),\ \text{if}\ s_{\perp,t} = 0\ \text{for}\ t \in [t_0, t_1] \tag{3.13}$$

The analysis is extended to the case where the predicted probability density function of the relative position is a sum of Gaussian densities. Simulation results show that this approach, compared with the Paielli and Erzberger model [24], is more sensitive with respect to the system parameters, and therefore may be able to distinguish better between safe and unsafe situations.

In [8], the authors provide analytical expressions for other performance metrics as well, such as the *Incrossing Rate* and the *Overlap Probability* of a two aircraft encounter. For further details the reader is referred to [8], or to the HYBRIDGE D2.1 under Work Package 2.

Other approaches include modeling of the prediction error as a scaled Brownian Motion [13], a multi-stage alerting concept based on Monte Carlo simulation [36], and a model where the predicted position error results from errors in the velocity and acceleration [7].

Comments: These methods aim to estimate the probability of conflict but don't provide specific conflict resolution advisories to the cockpit. A common assumption in all cases is that the prediction error is normally distributed.

3.2 A Probabilistic Conflict Detection and Resolution Algorithm

While most work on probabilistic models for CDR focuses on the estimation of prediction errors on the aircraft trajectory, very few of these provide specific routing instructions to the aircraft involved in the encounter. A decentralized algorithm for CDR has been presented in [27], and it is the most complete procedure of that type in literature. The authors consider two situations of conflict detection: mid-range conflict detection is used to provide centralized information to the ATC for a probable conflict whereas short-range conflict detection is performed on board and issues decentralized instructions to the pilots for a probable conflict within a time horizon of a few minutes. Randomized optimization is used in the first case to approximate the maximum of the PC in a mid-range time horizon (20 min.) and a closed form expression for the PC is given in the latter case, followed by a proposed decentralized conflict resolution algorithm. We give an outline of the second part of this work in the following paragraphs. A brief description of the first part can be found in [35].

The following stochastic kinematic model is considered for the motion of the two aircraft flying in the same altitude:

$$\begin{aligned} x^A(t) &= u^A t + \Sigma B^A(t) \\ x^B(t) &= \Delta x_0 + u^B t + R(\theta)\Sigma B^B(t) \end{aligned} \tag{3.14}$$

where θ is their relative heading angle, Δx_0 is their relative position at t=0, $x^i(t), u^i, i = A, B$ are their positions and speeds which are again assumed to be constant, $B^i(t), i = A, B$ are standard 2-D Brownian Motions, $R(\theta)$ is a rotation matrix and $\Sigma = diag(v_a, v_c)$, where v_a^2, v_c^2 the power spectral densities of the tracking errors in the along-track and cross-track directions respectively.

An appropriate coordinate transformation yields the following relative motion model for the two aircraft:

$$\Delta s(t) = \Delta s_0 + ut - \bar{n}(t) \tag{3.15}$$

where $\Delta s(t)$ is the relative position of the two aircraft at time t, Δs_0 their initial relative position, u their relative speed and $\bar{n}(t)$ a standard 2-D Brownian Motion starting at zero. Similar to [13], the motion of aircraft *A* is considered as a standard 2-D Brownian Motion starting at the origin and of aircraft *B* as a motion with constant velocity *u* starting at Δs_0 (Fig. 3.2). The circular conflict zone of radius $\rho = 5$ nmi around aircraft *B* is an ellipse in the new coordinates, moving along the relative velocity. A conflict occurs if the standard 2-D Brownian Motion $\bar{n}(t)$ starting at the origin ever hits the moving ellipse.

The distance of the origin from the line *h* along which *B* is flying, and the distance *a* of the initial position of *B* from the projection of the origin to the line *h* are given by:

$$x_d = \frac{|\Delta s_0^T R(\pi/2) u|}{\| u \|}, \quad a = -\frac{\Delta s_0^T u}{\| u \|} \tag{3.16}$$

Let *k* be the line vertical to *u* and passing through the center of the ellipse. The projected width 2*L* of the ellipse along *k* is:

$$L = \frac{\rho}{\lambda_1 \lambda_2} \sqrt{\frac{u_1^2 \lambda_1^2 + u_2^2 \lambda_2^2}{2(u_1^2 + u_2^2)}} \tag{3.17}$$

where λ_1, λ_2 are parameters related to the coordinate transformation performed previously.

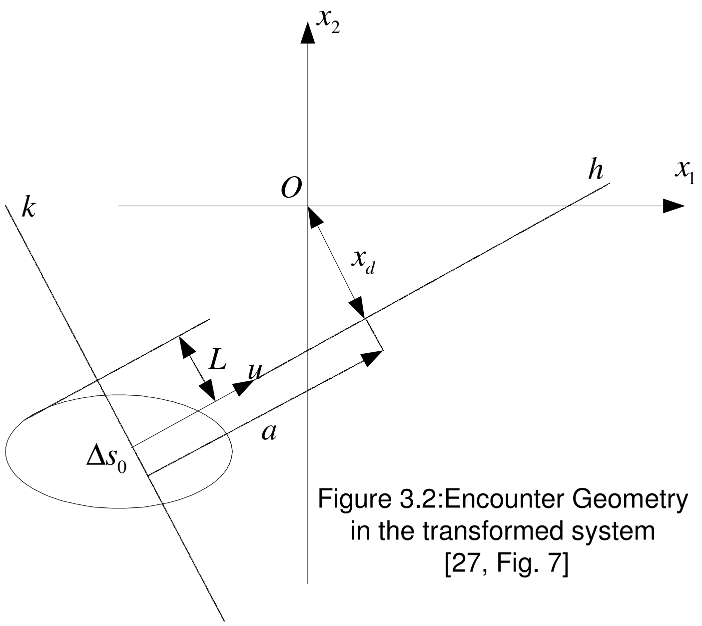

Figure 3.2:Encounter Geometry in the transformed system [27, Fig. 7]

The authors then use an analysis similar to that of [18] and [24] and conclude that the probability of conflict over an infinite time horizon can be approximated by:

$$P_C = Q\left(\frac{x_d - L}{\sqrt{t_0}}\right) - Q\left(\frac{x_d + L}{\sqrt{t_0}}\right) \tag{3.18}$$

where $Q(y) = \int_y^\infty \frac{1}{\sqrt{2\pi}} \exp(-z^2/2) dz$ and $t_0 = a / \| u \|$. An approximation of the probability of conflict over a finite time horizon is derived in the same way.

These results are then used in a decentralized conflict detection and resolution algorithm. Let a^A, a^B denote the current positions of two aircraft *A*,*B* respectively, b^A, b^B their destinations and u^A, u^B their velocities which have constant magnitude. For each aircraft, let θ_c denote its current heading, θ_d its destination heading and θ_g the direction of the negative gradient of P_C, i.e. the direction corresponding to its highest decrease. The heading parameters are updated every Δt time instants according to the following conflict resolution algorithm:

Given $x^A(k), x^B(k), u^A(k), u^B(k), \theta_c^i(k-1)$ compute $P_C(k)$.
for $i = A, B$ **do**
begin

Compute $\theta_d^i(k), \theta_g^i(k)$.

Set $\bar{\theta}^i(k) := P_C(k)\theta_g^i(k) + (1 - P_C(k))\theta_d^i(k)$.

Set $\theta_c^i(k) = \begin{cases} \bar{\theta}^i(k), \text{ if } |\bar{\theta}^i(k) - \theta_c^i(k)| < \beta \\ \theta_c^i(k-1) + \beta \operatorname{sgn}(\bar{\theta}^i(k) - \theta_c^i(k-1)), \textit{otherwise.} \end{cases}$

end

where β is the maximum turn angle at each update and $z(k) = z(k\Delta t)$. The aircraft tend to decrease the probability of conflict when $P_C(k)$ is high, whereas they aim to get nearer their destinations when $P_C(k)$ is low.

Monte Carlo simulations then prove the validity of the algorithm and an extension to the multiple aircraft case is proposed.

Comments: The result (3.18) is similar to that of the previous algorithms. In the proposed algorithm, aircraft choose their heading by decreasing the probability of conflict. The extension to *N* aircraft in [27] is rather informal and intuitive.

3.3 General Conclusions on Stochastic Methods for CDR

The methods described in the previous sections provide a summary for stochasticity-based approaches to CDR. General conclusions can be drawn after reviewing these works:

- The greatest amount of work so far has been devoted to conflict probability estimation and not to specific CDR algorithms.
- Some common assumptions are taken into account by most authors: the normal distribution of the prediction error, the disregard of the cross correlation of the predicted position errors of the two aircraft and the fact that

the velocities of the two aircraft are assumed to be constant throughout the encounter.
- There is no direct extension of the above methods to $N>2$ aircraft, apart from the informal discussion in [27].

It is obvious that stochastic based methods for CDR are rather incomplete, and less simplified approaches should be pursued.

Chapter 4

Alternative methods for CDR

Having overviewed the most standard stochastic and optimization-based methods for distributed CDR, we now make a discussion on some other approaches which are less common in the Air Traffic Control literature, yet they are very promising and provide motivation for future research. These methods have received considerable attention throughout the years in other fields, such as economics or robot navigation, but haven't been used to such extent in the aircraft collision avoidance case. In section 4.1 we describe a distributed artificial intelligence scheme for multiple agents based on principled negotiation presented in [34]. Section 4.2 is devoted to potential field methods for aircraft CDR.

4.1 Conflict Resolution based on Principled Negotiation

In [34], a method for multiple aircraft CDR is proposed based on a technique borrowed from distributed artificial intelligence: principled negotiation. The underlying method lies between the centralized coordination and non-cooperative distributed approaches. In the principled negotiation framework, each agent repeatedly tries to optimize an initial master plan according to his interests. If this action affects the interests of some other agent negatively, the latter issues a rejection and the other agent takes this into account in his new proposal. Therefore each agent takes the best initiatives for itself without affecting the mutual gain.

Each agent $i \in \{1,...,N\}$ has an action plan $a_i(t) \in A_i$, where A_i is the set of feasible action plans for agent *i*. The dynamics of the whole multi-agent system are characterized by the action profile *a*:

$$a(t) = (a_1(t),...,a_n(t)) \tag{4.1}$$

The consequences *c* of an action profile depend on the action profile, the initial state ω and the disturbances *v*, $c = c(a,\omega,v)$. The consequences of an action profile *b* are preferred to those of *a* by an agent *i* if they are better for its interests. This is encoded in the following:

$$c(b,\omega,v) \succ_i c(a,\omega,v) \tag{4.2}$$

where $\succ_i$ denotes the *preference relation* for agent *i*.

The authors distinguish between two types of agents: maximizers and satisfiers. The first aim to maximize a function of the consequences, called the *utility function* u_i. Between *a* and *b*, maximizer *i* prefers the action profile *b* if

$$u_i(c(b),\omega_{des}) > u_i(c(a),\omega_{des}) \tag{4.3}$$

where ω_{des} is the desired final state. Satisfiers want the consequences to keep some quantities satisfactory. This is encoded in the *satisfying function* s_i. The action plan *a* is acceptable by satisfier *i* if

$$s_i(c(a)) > s_{\min} \tag{4.4}$$

where $s_{\min}$ is the vector of minimum acceptable values for agent *i*. When the considered system is stochastic, each agent can only make estimates of the consequences and the corresponding preference functions of an action plan. In this case equations (4.3),(4.4) become

$$\begin{aligned} &E_i\{u_i(c(b))\} > E_i\{u_i(c(a))\} \\ &E_i\{s_i(c(a))\} > s_{\min} \end{aligned} \tag{4.5}$$

where $E_i\{\}$ is the expectation operator for agent *i*.

When an agent has no knowledge of the others' goals and interests, he proposes options *b* of the form (4.5). To be accepted, this option must not decrease the utility function of the other agents (without loss of generality, we assume that all agents are maximizers):

$$E_j\{u_j(c(b))\} \geq E_j\{u_j(c(a))\}, \forall j \in N \setminus \{i\} \tag{4.6}$$

A coordinator is needed to ensure that any agents influenced by the proposed profile have the chance to reject or accept it. When an agent has knowledge of the other agents, it plans its actions not only according to (4.5), but also with an expectation that all other agents will not at least be affected by these actions:

$$E_i\{u_j(c(b))\} \geq E_i\{u_j(c(a))\}, \forall j \in N \setminus \{i\} \tag{4.7}$$

The framework described above is then applied to a 2-D *n* aircraft encounter. The action plan of each agent is defined as its position in time. The coordination criterion is the maintenance of the separation minimum for each pair of aircraft, i.e.

$$\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} > D, \forall i, j \in N, i \neq j \tag{4.8}$$

Each agent has knowledge of the action plans of all agents within radius *r* from its center of gravity. The authors consider three optimization schemes: a distributed optimization scheme with no cooperation between agents ($r = 0$), a centralized scheme ($r \to \infty$), and a distributed optimization scheme based on principled negotiation. Simulations are performed in each case and show the benefits of the proposed method in terms of safety and economy.

Comments: The developed theory can be viewed as an intermediate between complete non-cooperation and fully centralized approaches. The definition of the *preference relations* (i.e. negotiation criteria) requires more specification and is an interesting topic for further research.

4.2 Potential Field Approach to CDR

Potential field methods have been widely used in mobile robot navigation in the last two decades. However they have not been applied at that extent in aircraft conflict detection and resolution, mainly because of the input constraints imposed in the aircraft kinematic model. An excellent reference on the application of various potential field methods on aircraft CDR is [38], which is similar to the discussion on coordination between multiple mobile vehicles in [39]. The discussion here is based on [17], which to our knowledge is the most complete approach of that type. The proposed distributed algorithm does not always generate flyable trajectories but it provides prototype guidelines for coordinated maneuvers between multiple aircraft.

In a planar situation each aircraft *i* is considered to have a protected circle of radius r_i around its center of gravity and let $q_i = (x_i, y_i)$, $q_{d_i} = (x_{d_i}, y_{d_i})$ denote its current and destination configuration respectively. The latter is encoded in the attractive potential function

$$U_a(q_i, q_{d_i}) = \frac{1}{2}\| q_i - q_{d_i} \|^2 \tag{4.9}$$

The corresponding force generated by the negative gradient of U_a is of course

$$F_a(q_i, q_{d_i}) = -\nabla U_a(q_i, q_{d_i}) = -(q_i - q_{d_i}) \tag{4.10}$$

The following repulsive field ensures collision avoidance between aircraft *i* and *j*:

$$U_r(q_i, q_j) = \begin{cases} -\frac{1}{2\delta_{r_j}}(r_{ij} - (r_j + \delta_{r_j}))^2, \; if \;\; r_j \le r_{ij} \le r_j + \delta_{r_j} \\ 0, \; otherwise \end{cases} \tag{4.11}$$

where r_{ij} is the distance between the two aircraft, and δ_{r_j} is the influence zone of the repulsive field of the j^{th} aircraft. The corresponding repulsive force is $F_r(q_i, q_j) = \nabla U_r(q_i, q_j)$. Finally, in order to prevent deadlocks in perfectly symmetric encounters in a conflict resolution procedure, the following vortex field is constructed around each agent:

$$F_v(q_i, q_j) = \pm \begin{bmatrix} \frac{\partial U_r(q_i, q_j)}{\partial y} \\ -\frac{\partial U_r(q_i, q_j)}{\partial x} \end{bmatrix} \tag{4.12}$$

The sign of F_v defines the direction to which aircraft *i* and *j* should head in a conflict encounter. The proposed dynamic model for aircraft *i* is then

$$\dot{q}_i = \frac{F_a(q_i, q_{d_i})}{\| F_a(q_i, q_{d_i}) \|} + \sum_j (k_{r_i} F_r(q_i, q_j) + k_{v_i} F_v(q_i, q_j)) \tag{4.13}$$

Two types of conflict resolution maneuvers are considered for simulation purposes: *overtake* and *head-on* maneuvers. In the multiple aircraft case the direction of the vortex field serves as a coordinator between the aircraft.

An extension of this work can be found in [11], where the authors provide proofs of the kinematic safety of the underlying method for up to three agents, without again taking into account velocity constraints.

Comments: The algorithm cannot guarantee collision avoidance when input constraints are taken into account, especially in the multiple aircraft case. Input constraints are the main reason for the lack of use of potential field methods in aircraft CDR.

Chapter 5

Conflict Detection and Resolution based on Navigation Functions

Having overviewed the various decentralized CDR methods in literature, in this chapter we present part of the work held in the Control Systems Lab of NTUA as part of the HYBRIDGE effort. We aim to apply these methods to the decentralized aircraft CDR problem. The work presented in section 5.2 is based on [22],[37]. In section 5.1 the main conclusions of the discussion in the previous chapters are summarized.

5.1 General Conclusions on Distributed CDR Methods

The discussion on the various distributed CDR methodologies in literature given in the previous chapters has lead to the following useful observations:

- Optimization techniques are a classic and attractive mathematical tool to deal with such problems. However, most methods are computationally expensive when dealing with more than two aircraft and do not guarantee to have a solution.
- Stochastic based methods are to a large extent devoted to the computation of the probability of conflict between a pair of aircraft and only the work described in section 3.2 provides specific guidelines for decentralized conflict resolution.
- The potential field approach presented in section 4.2 cannot deal with the input constraints required by the control design.

The drawbacks of the existing approaches highlighted above reveal the need for more powerful mathematical methods for decentralized CDR both in the deterministic, as well as in the stochastic domain.

5.2 Navigation Functions-Proposed Research Objectives

Navigation functions are real valued maps realized through cost functions, whose negated gradient field is attractive towards the goal configuration and repulsive with respect to obstacles. This tool has been used in [22] for centralized navigation of multiple vehicles, whereas a decentralized extension has been presented in [37]. In the following paragraphs, we briefly describe the main features of these works.

In [22] the following situation is assumed: m mobile vehicles move in a workspace $W \subset R^2$. Each vehicle R_i, $i = 1,...,m$ occupies a disc in the workspace: $R_i = \{q \in R^2 : \|q - q_i\| \leq r_i\}$, where q_i the position of the center of gravity of the vehicle. The configuration of the vehicles is given by $q = [q_1^T,...,q_m^T]^T$. Let q_d denote the desired destination configuration. A navigation function is a map $\varphi : F \rightarrow [0,1]$, where $F \subset R^n$ is an analytic manifold with boundary, that satisfies the following properties:

1. It is analytic on F,
2. It has only one minimum at $q_d \in \overset{o}{F}$,where $\overset{o}{F}$ denotes the interior of F,
3. Its Hessian at all critical points (zero gradient vector field) is full rank, and,
4. $\lim_{q \to \partial F} \varphi(q) = 1$.

In the centralized setting of [22], the proposed control law is given by

$$u = -K\nabla\varphi(q) \tag{5.1}$$

The considered navigation function is

$$\varphi = \sigma_d \circ \sigma \circ \hat{\varphi} = \left(\frac{\gamma}{\gamma + G}\right)^{1/k} \tag{5.2}$$

where $\sigma_d = x^{1/k}$,$\sigma = \frac{x}{1+x}$ and $\hat{\varphi} = \frac{\gamma}{G}$ the cost function, for which $\gamma^{-1}(0)$ denotes the desirable set (i.e. the goal configuration) and $G^{-1}(0)$, the set that we want to avoid (i.e. collisions with other vehicles). The function *G* serves as an indicator of a specific collision scheme, since it tends to zero when a collision scheme tends to occur and increases as the danger of the collision situation fades.

The cost function $\hat{\varphi} = \frac{\gamma}{G}$ has the property that the set of its critical points and that of φ coincide and the (Morse) index of each critical point is identical. Hence, results for the function φ can be derived by examining the simpler function $\hat{\varphi}$.

After the necessary terminology (i.e. the definition of the function *G*), the work of [22] (and subsequently, of [37]) proceeds with the proof of the following propositions:

Proposition 1: *If the workspace is valid, the destination point q_d is a non–degenerate local minimum of φ.*

Proposition 2: *If the workspace is valid, all the critical points of φ are in the interior of the free space.*

Proposition 3: *For every ε>0, there exists a positive integer N(ε) such that if k>N(ε) then there are no critical points of $\hat{\varphi}$ in $F_1(\varepsilon)$, where $F_1(\varepsilon)$ denotes the set away from collisions.*

Proposition 4: *For any valid workspace, there exists an $\varepsilon_0 > 0$ such that $\hat{\varphi}$ has no local minimum in $F_0(\varepsilon)$, as long as $\varepsilon < \varepsilon_0$, where $F_0(\varepsilon)$ denotes the set near collisions.*

These propositions establish that goal configurations are achievable (not any collisions at the target) and that there will always be a direction of movement decreasing the potential function. Hence there will always be a direction that drives the vehicles to the desired destination without collisions.

In [37], a decentralized extension of the control law (5.1) is proposed. In this approach, each vehicle has knowledge only on its own goal configuration. The proposed control law for each robot $i = 1,...,m$ is:

$$\dot{q}_i = -K_i \frac{\partial \varphi_i(q)}{\partial q_i} \tag{5.3}$$

The decentralized navigation function of [37] is given by

$$\varphi_i(q) = \frac{\gamma_d(q_i) + f(G(q))}{\left(\left(\gamma_d(q_i) + f(G(q))\right)^k + G(q)\right)^{1/k}} \tag{5.4}$$

where $\gamma_d(q_i) = \|q_i - q_{id}\|^2$ is the squared metric of the current vehicle's configuration q_i from its desired destination q_{id}.

The inclusion of the function $f(G)$ encodes some form of cooperation between the moving agents. Specifically the f function wont let the agent approach too close to another moving agent, which is related to setting lower bounds on the minimum acceptable distance between the two. For more details, the reader is referred to [37].

The previous discussion reveals some important conclusions, which provide motivation for further research:

1. The Navigation Function methodology has been proven to be a powerful and effective mean of mobile vehicle navigation, both in the centralized, as well as in the *decentralized* domain, which is of main importance in WP6. This has found direct application to the robot collision avoidance problem, where it produces *provably conflict-free* trajectories.

2. An extension to the decentralized aircraft CDR problem is both interesting and appealing but it is far from straightforward. This is due mainly to the following issues:
   - Aircraft and especially civil ones have strict constraints regarding their velocities. Hence, similarly with [17], input constraints have to be taken into account.
   - The aircraft kinematic model takes into account its angular velocity and so is certainly non-holonomic. The simple holonomic model used in [22],[37] must be extended to that case.
   - Uncertainty factors must also be taken into account.
   - The extension of this methodology to Non-Level, i.e. 3-dimensional, flight is an important factor, which has not been addressed yet.

- The current method assumes a fixed number of agents. Therefore, it must be reformulated in order to cope with the dynamical change in the number of agents in the conflict resolution procedure.

These conclusions are the main topics of current research efforts. Specifically, we aim to apply the idea of navigation functions to decentralized navigation of multiple aircraft with input constraints. We also aim to extend this method to the non-planar case. Stochastic aspects are the goal of future research.

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APPENDIX

CDR Concepts in the NASA/NLR Free Flight Project

One of the early started developments of distributed CDR concepts was within the NASA/NLR Free Flight Project. This is an effort that started officially in January 1997 as a five-year program, and has now been expanded with another five-year period. In this project, Free Flight is treated as a progressive reassignment of the aircraft separation procedures from the ATC to the cockpit, incorporating two main elements: Airborne Separation for a substantial part of the flight and Direct Routing both horizontally and vertically. In the following we make a brief description of the main distributed CDR methodologies used in this project and try to relate them to the CDR theory presented in the main part of our report. The discussion is based on Jacco Hoekstra's PhD thesis, which can be found at http://www.nlr.nl/public/hosted-sites/freeflight/main.htm.

The researchers distinguished between two modules in the CDR procedure: conflict detection and conflict resolution. Two types of criteria were taken into account in selecting the CDR method. Firstly, the criteria resulting from the operational concept (state (position and velocity)-based CDR, no priority rules and no explicit co-ordination) and secondly, some general criteria for the validation of the quality of each method. These included safety, efficiency, human factors and technological criteria.

For the conflict detection module several methods in literature were proposed and examined: state-based conflict detection, enhanced state (position, velocity and mode control panel) based conflict detection and route based (flight plan) based conflict detection. The operational concept criteria allowed only the use of the first methodology. The state-based conflict detection is implemented in a straightforward way: look at the current state (position and velocity) vector and calculate the predicted minimum distance with the other aircraft. If this is smaller than the separation minimum, an alert is triggered to the cockpit, whose operation switches on immediately to the conflict resolution module. A conflict is detected whenever the predicted minimum distance within the lookahead time is smaller than the required minimum separation distance. The lookahead time was set at five minutes.

It is obvious that the aforementioned methodology is clearly related to the stochastic CDR methods discussed in chapter 3 of the main part of our report, and in particular to R. Irvine's work [16]. The calculation of the minimum distance can be made either deterministically or stochastically.

For the conflict resolution module there were four alternatives: no resolution procedure (leaving it up to the pilot to maneuver), geometrical methods, numerical optimization methods and genetic algorithms. Since no maneuvering priority rules were considered, the first alternative was rejected immediately. Numerical optimization methods, which are directly related to the theory presented in chapter 2 of our report, were also rejected as they were only applicable in a flight-plan based CDR procedure, whereas only state-based CDR was taken into account by the operational concept. Finally, genetic algorithms were rejected, mainly because of the computational complexity of the algorithms and the fact that high computing power in the cockpit is not as common as on desktop computers. A genetic-type methodology is discussed in section 4.1 of our report.

That left geometrical conflict resolution methods. The researchers compared between rule-based methods and potential field methods and the latter was their final choice. Rule-based methods included altitude step maneuvers and variations on the TCAS maneuvers, i.e. climb and descend maneuvers. The main disadvantage of this approach was that it required extra hardware for resolution co-ordination and extra bandwidth. The finally chosen method is similar to the potential field approach described in section 4.2 of our report. Each aircraft is equipped with a sum of repulsive forces from all other aircraft and an attractive force towards its desired destination. In that way, a vector is determined which maintains separation with other aircraft and brings the aircraft to its destination. The researchers have developed a less simplistic and more realistic version of this method. For more details, the reader is referred to the link of the first paragraph.

The preceding discussion reveals the fact that the gap between the current theory and practice is not as big as it might have been thought. It is clear that the mathematical analysis presented in the main part of the report can and has been used in practical situations. It is the goal of the future to further apply the theoretical advancements to realistic scenarios in ATM.

CDR Concepts in FREER Flight Project of EEC (Bretigny)

In this part of the appendix, we make a short discussion on the work on distributed CDR developed within the Eurocontrol project FREER (Free-Route Experimental Encounter Resolution). This was an initiative that started in late 1995 to investigate the feasibility of the concept following which ATC functions could be delegated to the flight deck to allow more freedom of movement to airspace users, support the implementation of Free Flight, Free-Route and User-Preferred Route concepts, and to involve airspace users in the ATM loop. The information is based on the document list of the project that can be found at http://www.eurocontrol.fr/projects/freer/publications.htm

Based on the spectrum of autonomy that can be granted to airspace users, FREER researchers considered in the early stages of the project three generic ATM operational modes: Ground-based Centralised Control, Ground-Air Co-ordinated Control, and Airborne Autonomous Control. Taking these generic nodes into account the FREER researchers approached the problem of distributed ATM in two different manners: a) in a fully decentralized fashion, under the framework of *Airborne Autonomous Control mode* (FREER-1), where ATC and Trajectory Management functions are fully delegated to the flight deck for operations in low-density airspace and b) in a partially decentralized fashion, under the framework of *Transition from the Centralised Control mode to the Ground-Air Coordinated Control mode* (FREER-2), where ATM activities are only partially delegated to the FREER-1 capable aircraft to operate in high-density airspace. Here, we discuss the CDR concepts in the FREER-1 concept, which is more closely related to the spirit of our report.

In FREER, the conflict detection problem is defined as the process of detecting the portions of the trajectories during which the distance between aircraft violates a separation standard. FREER-1 addresses the issue of uncertainty handling by considering a geometrically deformable volume defined from the aircraft position. The distances from the aircraft to the surfaces of the volume are used to represent the uncertainties associated with the trajectory and flight phases. The distance from the aircraft to the left surface could be larger than that on the right during a certain period of time. Actually, the initial idea for this representation was to associate a Gaussian distribution to a trajectory. The likelihood can be then calculated from the integral of the intersection (sum) of the Gaussian's of two trajectories. This is clearly related to the work of Paielli and Erzberger [24], discussed in chapter 3 of the current report. In this approach, a constrained minimisation algorithm for computing conflict using an interval Newton method on physically based trajectories of objects was chosen.

The conflict resolution problem consists of finding a path for the aircraft to avoid a set of forbidden zones, or no-go zones, which have been evaluated in the conflict detection procedure laterally, vertically, or longitudinally. The selection of the trajectories involved minimization of the extra-cost associated with a deviation of a trajectory that avoids conflicts. This is similar to the approach of chapter 2 of our report.

CDR Concepts, which have initially been evaluated on Safety

In this part of our report, we make a brief review of the ATM approaches evaluated within the CARE-ASAS-3 work for Eurocontrol. The action plan of this project involved development of operational procedures towards a progressive transport of the separation maintenance responsibility between aircraft from the ATCs to the cockpit. In particular, the main features of Activity 3 of this project, regarding Airborne Separation Minima, are investigated. These are directly related to the distributed mathematical models presented in this report and therefore are of great interest for the future purposes of HYBRIDGE WP6.

The Work Package 3 of the CARE-ASAS Activity 3 aimed at a first estimation of the safe separation minimum between aircraft. This was applied into two typical ASAS scenarios: Autonomous Operations (AO) and Co-operative Separation Assurance (CSA). From these, the first one is clearly related to the orientation of this report.

The autonomous aircraft application is defined in Free-Flight Airspace, which is a segregated airspace where only suitably equipped aircraft are accepted. Pilots are allowed to fly their user-preferred routes while they lie within the boundaries of the Free-Flight Airspace. In the Free-Flight Airspace, the cockpit is responsible for self-separation in accordance with the applicable airborne separation minima. Conflict Resolution is performed according to specific priority rules defining which of the aircraft in conflict has to maneuver. The mathematical model adopted for this approach was based on the Traffic Organization and Perturbation Analyzer (TOPAZ) methodology, which is directly related to the theory of stochastic CDR methods presented in Chapter 3 of the current report. The subsequent task was to perform accident risk assessment for this mathematical model. Several differences between the AO approach and the accident risk model were identified. These included numerical approximation assumptions, parameter value assumptions and model structure assumptions.

The Co-operative Separation Assurance application taken into account was the time-based sequencing application. This involves the delegation of separation responsibility from the Air Traffic Controller to the flight-crew. The flight crew of the delegated aircraft is responsible for complying with respect to the time-based separation objective defined by the controller with respect to the designated aircraft in accordance with the airborne separation minima. In this case however, the controller is still in charge of ensuring ATC separation between both aircraft involved and surrounding traffic. The theory of stochastic CDR methods presented in Chapter 3 of the current report is applicable in this case as well.

Further details can be found in the CARE-ASAS website, and in particular in the documents list therein:
http://www.eurocontrol.int/care/asas/documentation/reference_document_list.htm.