Aircraft and Weather Models for Probabilistic Collision Avoidance in Air Traffic Control

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Abstract

We propose a discrete time probabilistic model for predicting the future position of airliners, based on information about their current positions and flight plans. The model is used to derive an algorithm for detecting possible conflicts between aircraft, situations where the aircraft may come closer than a certain distance to one another with high probability.

Keywords: Air Traffic Management; Air Traffic Control; probabilistic collision avoidance.

1 Introduction

Despite technological advances in navigation, communication, computation and control, the Air Traffic Management (ATM) system is still, to a large extent, built around a rigidly structured airspace and a centralised, mostly human-operated system architecture. The increasing demand for air travel is stressing current ATM practices to their limit. Moreover, projections indicate that air traffic could double over the next ten years. This is likely to cause both safety and performance degradation in the near future, and place an additional burden on the already overloaded human operators. It is believed that by increasing the level of automation, the efficiency of ATM can be improved and the tasks of human operators can be simplified. This will allow them to handle the increased demand in air traffic in a more reliable way, enhancing the level of safety over the current system.

The primary concern of all advanced ATM systems is to guarantee *safety*. Safety is typically quantified in terms of critical situations, for example, *conflict* situations where two aircraft come closer than a certain distance to one another. In this context, the main tasks involved in ATM safety analysis are *conflict detection* (estimating the criticality of a given situation) and *Conflict resolution* (designing algorithms for preventing or resolving safety critical situations).

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In this paper we propose a new method for conflict detection at the Air Traffic Control (ATC) level of the ATM system. The main features of the proposed method are:

Model Based. A model of the aircraft motion from the point of view of ATC will be used to predict the positions of aircraft in the future.

Probabilistic. The model will produce a probability distribution for aircraft positions. This will allow us to incorporate uncertainty (due primarily to wind) in our predictions. Conflict detection will involve estimating the probability of conflict.

Mid-range. Conflict detection will be carried out over horizons of the order of tens of minutes.

Level Flight. The development is carried out in two dimensions, assuming level flight. We believe that the extension of the algorithms to three dimensions will be relatively easy conceptually, but may be significantly harder computationally.

We start by putting our work in context with respect to the state of the art in ATM research (Section 2). We then present the model we propose for modelling the flight of airliners from the ATC point of view (Section 3) and discuss how it can be incorporated in a probabilistic conflict detection algorithm (Section 4). We conclude with a brief discussion of our current research on this problem (Section 5).

2 Background

Conflict detection and resolution for ATM relies on methods for predicting the position of aircraft in the future, based on measurements about their current position and information about their intents (e.g. their flight plans).

One of the key elements in the prediction is modelling the uncertainty inherent in aircraft motion, due to the wind, the responses of human operators, and the errors in tracking, navigation, and control. Prediction schemes can be classified in three categories, according to the method they use for taking this uncertainty into account. The *nominal* approach assumes that the aircraft will continue to move along their current path or flight plan. This prediction method is fairly simple and widely used in practice; for example, it is the prediction method used by the Traffic alert and Collision Avoidance System (TCAS) [1]. However, it is not robust, since uncertainty is neglected, and may lead to optimistic predictions. The *worst case* approach, on the other hand, assumes that the aircraft will follow the worst possible path within the allowable uncertainty set. This approach (which is adopted in [2] for example) is essential for establishing absolute performance guarantees. It tends, however, to be too conservative in practice. Finally, in a *probabilistic* approach the uncertainty is taken into account by considering the ensemble of sample paths generated by a stochastic model of the motion of the aircraft, and assessing the criticality of the situation in terms of the "probability of conflict".

A thorough overview of the state of the art for all of these methods can be found in [3]. The present paper takes a probabilistic approach to the problem. The reason for this choice is that it avoids the conservativeness of the worst-case approach, but is more robust than the nominal approach.

A number of probabilistic models have already been developed to capture different aspects of ATM. One of the most popular ones is the model developed by NASA [4], as part of the Centre TRACON Automation System (CTAS). This model is designed to operate at the ATC level and takes a centralised view of the ATM process. The motion of the aircraft is characterised by a probabilistic deviation of the aircraft position with respect to the deterministic flight plan. The probabilistic deviation takes the form of a Gaussian distribution, whose covariance matrix grows in time, to reflect the fact that the validity of our predictions decreases the further we try to project the aircraft position into the future.

This model has a number of drawbacks that limit the accuracy of predictions based on it. Rather than trying to capture the evolution of individual trajectories, the model attempts to match the ensemble behaviour of all the trajectories of the aircraft. This leads to predictions that overlook fundamental facts such as the statistical correlation between positions of the same aircraft at different points in time, the correlation among the positions of nearby aircraft, etc. Here we propose a new probabilistic model that succeeds in alleviating most of these drawbacks (Section 3).

A number of different methods have also been proposed for computing the probability of conflict for two aircraft encounters. A closed-form expression for the probability of conflict for level flight is derived in [4], under a number of simplifying assumptions. In spite of its simplicity, which makes it very attractive for on-line implementation, this method does not allow one to formally analyse the performance of the algorithm. Moreover, the exact interpretation of the results obtained by applying the closed-form formula is unclear when, as is often the case, the simplifying assumptions are not satisfied. In [5], Monte Carlo simulation is used to compute the probability of conflict. This approach does not require particular assumptions and can be applied to many different scenarios. However, it is also not amenable to formal analysis. Moreover, it is computationally intensive and therefore may not be suitable for on-line implementation.

In earlier work [6] the authors demonstrated how randomised algorithms can be used to alleviate the shortcomings of both these approaches. In addition to being efficient computationally, the proposed algorithms provide explicit (albeit probabilistic) performance guarantees, and allow one to optimise the tradeoff between accuracy and computation time. In Section 4 we extend this approach to the probabilistic model developed in Section 3.

3 Modelling

Our model attempts to predict the position of aircraft over a horizon of T in the future (typically, $T = 20$ minutes). It deals with level flight and comprises the following components:

Flight Plan. A sequence of way points and speeds that determine the nominal path of the aircraft.

Aircraft Kinematics. A discrete time system to model the horizontal movement of aircraft.

Flight Management System. A controller for the kinematic model, whose gains determine how the FMS tracks straight line paths.

Nominal Wind Model. A function (possibly given as a periodically updated look-up table) providing the nominal wind at each point in the airspace.

Stochastic Wind Perturbation. A two dimensional random field to capture the stochastic deviation between the nominal and actual wind.

The rest of this section is devoted to filling in the details for all these components.

3.1 Flight Plan

We assume that during the time horizon, T, of interest the aircraft is flying at a constant altitude, following a flight plan defined by a sequence of way points and air speeds $\{(O_i, v_i)\}_{i=0}^{N}$ with $O_i \in \mathbb{R}^2$ and $v_i \in \mathbb{R}_+$. The coordinates of the way points are typically measured in nautical miles and are assumed to be given in a global

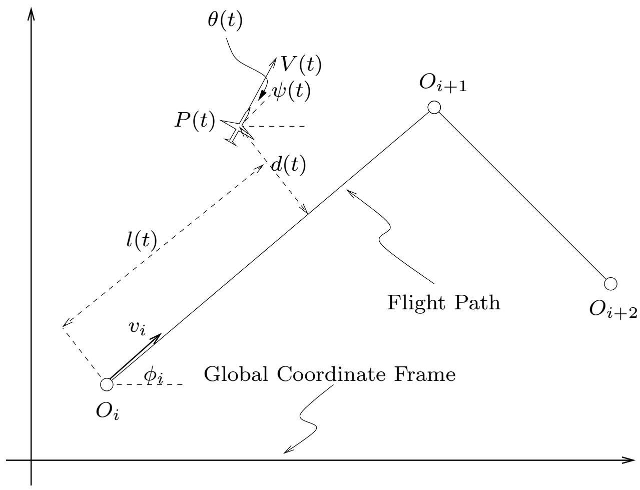

Figure 1: A segment of the flight plan and the related notation.

coordinate frame. The speed is typically measured in nautical miles per hour.

Two consecutive way points, O_i and O_{i+1}, define a straight line segment $O_{i+1} - O_i$ with slope ϕ_i in the global coordinate frame (Figure 1). This straight line segment constitutes the *nominal path* that the aircraft is supposed to follow when travelling from way point O_i to way point O_{i+1}. v_i is the *nominal speed* of the aircraft when moving between these way points. We will use the term *flight plan* to refer to the sequence of way points and speeds $\{(O_i, v_i)\}_{i=0}^{N}$, and *flight path* to refer to the sequence of straight line segments defined by the flight plan in $\mathbb{R}^2$.

Because of the uncertainty inherent in the path tracking process, it is likely that at any point in time the aircraft will not be exactly on the flight path. The role of the flight management system is to get the aircraft to track the flight path as closely as possible. Let $P(t) \in \mathbb{R}^2$ denote the position and $\psi(t)$ the heading of the aircraft (defined here as the direction of the velocity) at time $t \in [0, T]$ in the global coordinate frame. The deviation of the aircraft from the flight path can be computed using a change of coordinates. Let us consider the flight path segment i. Denote by $R(\phi_i)$ the rotation matrix

$$R(\phi_i) = \begin{bmatrix} \cos(\phi_i) & \sin(\phi_i) \\ -\sin(\phi_i) & \cos(\phi_i) \end{bmatrix},$$

and define

$$\begin{aligned} X(t) &= R(\phi_i)(P(t) - O_i) \\ \theta(t) &= \psi(t) - \phi_i. \end{aligned} \tag{1}$$

If we denote by $l(t)$ the projection of $P(t)$ along the flight path and by $d(t)$ the deviation from the flight path (see Figure 1), one can verify that $X(t) = [l(t) \;\; d(t)]^T$.

3.2 Aircraft Kinematics

Fix a sampling time interval $\Delta > 0$. The aircraft motion can be described through the following discrete time kinematic model

$$\begin{aligned} P(t+\Delta) =& P(t) + \begin{bmatrix} \cos(\psi(t)) \\ \sin(\psi(t)) \end{bmatrix} v_i \Delta + W(t, P(t))\Delta \\ &+ N(t, P(t)), \\ \psi(t+\Delta) =& \psi(t) + \omega(t)\Delta, \end{aligned} \tag{2}$$

where $(P(t), \psi(t)) \in \mathbb{R}^3$ is the aircraft position and heading (system state), $\omega(t) \in \mathbb{R}$ is the rate of change of the heading (control input), $W(t, P) \in \mathbb{R}^2$ is the nominal wind velocity (a measured, deterministic disturbance), and $\{N(t, P) \in \mathbb{R}^2\}$ is the wind random field (an unmeasured, stochastic disturbance).

The model rests on a number of underlying assumptions. For example, the air speed of the aircraft is assumed to be constant and equal to $\|V(t)\| = v_i$. This assumes that the aircraft adjusts its heading but not its speed in response to deviations from the flight plan. This assumption is reasonably realistic for aircraft with 3D FMS, where the FMS is only concerned with getting to a way point ignoring timing constraints. We also assume that the air speed of the aircraft is bounded in a certain range, dictated by ATC practice, limitations of the air frame and engines, and the need to generate enough lift. Finally, to ensure that it is possible to design a stable FMS to track the flight path, we assume that the wind speed does not exceed the aircraft air speed.

3.3 Flight Management System

We assume a 3D FMS whose main function is to adjust the aircraft heading in order to track the flight path, without trying to meet timing requirements. More specifically, the FMS measures the path tracking error, $d(t)$, and the heading, $\psi(t)$, and uses their values to select $\omega(t)$ in an attempt to keep $d(t)$ small. We propose to model the FMS by the following linear, dynamic, feedback controller:

$$\begin{aligned} s(t+\Delta) &= s(t) + \Delta d(t), \\ \omega(t) &= -k_1 d(t) - k_2(\psi(t) - \phi_i) - k_3 s(t). \end{aligned} \tag{3}$$

Since the conflict detection computations in Section 4 are based on a linearisation of the model about a nominal trajectory they are easier to carry out in coordinates aligned with the flight path. If we let

$$\begin{aligned} \tilde{W}(t, X) &= R(\phi_i) W\left(t, R(\phi_i)^{-1} X + O_i\right) \\ \tilde{N}(t, X) &= R(\phi_i) N\left(t, R(\phi_i)^{-1} X + O_i\right) \end{aligned}$$

the closed loop system modelling the aircraft motion between way point O_i and way point O_{i+1} becomes

$$\begin{aligned} X(t+\Delta) =& X(t) + \begin{bmatrix} \sin(\theta(t)) \\ \cos(\theta(t)) \end{bmatrix} v_i\Delta + \tilde{W}(t, X(t))\Delta \\ &+ \tilde{N}(t, X(t)) \\ \theta(t+\Delta) =& \theta(t) - k_2\Delta\theta(t) - k_3\Delta s(t) \\ &- k_1\Delta \begin{bmatrix} 0 & 1 \end{bmatrix} X(t) \\ s(t+\Delta) =& s(t) + \Delta \begin{bmatrix} 0 & 1 \end{bmatrix} X(t), \end{aligned} \tag{4}$$

where $\theta(t)$ is defined in (1).

3.4 Modelling Turns

To facilitate the analysis we assume that the turn from one segment to the next happens when the projection, $l(t)$, of the aircraft position along the direction ϕ_i of the current segment of the flight plan first satisfies

$$l(t) \geq \|O_{i+1} - O_i\|.$$

We further assume that the turns between one segment and the next are instantaneous. This type of turning makes the analysis of the model somewhat simpler and is easy to implement in simulation, by switching the values of the parameters from O_i, ϕ_i, v_i to O_{i+1}, ϕ_{i+1}, v_{i+1} when the aircraft turns. We conjecture that this is also reasonably realistic from the point of view of the air traffic controller.

The resulting model is a nonlinear, stochastic, hybrid system. The discrete states of the system are the segments of the flight plan and are identified by the triple (O_i, ϕ_i, v_i). The continuous state corresponds to the aircraft position, heading, and the state of the FMS, i.e., $l(t)$, $d(t)$, $\theta(t)$, $s(t)$. The dynamics within each discrete state (O_i, ϕ_i, v_i) is governed by equation (4), which depends on the discrete state through the nominal speed v_i. Evolution can go on in each discrete state (O_i, ϕ_i, v_i) as long as $l(t) < \|O_{i+1} - O_i\|$. Transition between discrete states are "guarded" by conditions of the form $l(t) \geq \|O_{i+1} - O_i\|$. The continuous state is then reset according to equation (1). Notice that in the $X - \theta$ coordinates, the state of the system will undergo a jump at the turn points, since a change in ϕ_i and O_i will induce an instantaneous change in X and θ.

3.5 Nominal Wind Model, $W(t, P)$

Strictly speaking the model proposed here could operate without any information about the nominal wind. In this case the term $W(t, P(t))$ can be set to zero, and the wind can be assumed to be entirely an unknown random disturbance that enters the model through $N(t, P(t))$. This assumption is quite common in the literature; for example, the model of [4] does not include any information about the nominal wind. Of course the more a-priori information about the nominal wind we incorporate in the model, the more accurate the predictions based on the model are going to be. Our current assumption is that the nominal wind data will consist of a look up table that returns a vector $W(t, P)$ for different times t and different points P in the airspace. Data like this is made available both to the air traffic controllers and is updated periodically.

3.6 Stochastic Wind Perturbation, $N(t, P)$

We model $N(t, P)$ using a two dimensional random field, i.e. a family of random variables taking values in $\mathbb{R}^2$, defined on a common probability space and indexed by the 3-dimensional parameter vector (t, P). Motivated by realistic considerations (and to keep the analysis tractable) we assume that the wind random field is Gaussian, stationary, zero mean and isotropic. These assumptions imply that all finite dimensional distributions of $N(t, P)$ are Gaussian, and for all t_1, t_2, P_1, P_2, $E[N(t_1, P_1)] = 0$ and $E[N(t_1, P_1)N(t_2, P_2)^T] = R(|t_1 - t_2|, \|P_1 - P_2\|)$.

Considerable effort was made to form an idea of what the covariance function $R(\cdot, \cdot)$ looks like and obtain typical values for the variances and other parameters. This information proved to be surprisingly elusive. Experts in ATC, in dispersion of pollutants and in turbulence were consulted, but apparently none of these disciplines directly addresses the problem in question. Some information is available in the meteorology literature [7], but not at the level of detail that one would hope for. We are currently focusing on the Dryden-Kolmogorov atmospheric turbulence model [8] as a possible source of information about these parameters.

4 Conflict Detection

Conflict detection using the model proposed here is complicated because the model is nonlinear and the nominal wind and correlation structure of the random field depend on the state of the system. The conflict detection algorithm proposed in this section attempts to address these difficulties by systematic approximations. In particular, we propose to simulate the nonlinear model to generate a *nominal trajectory* for each aircraft. We then linearise the nonlinear model around the nominal trajectory and use the nominal trajectory instead of the state in the nominal wind look-up table and the correlation function of the wind random field.

The algorithm consists of a number of steps. The entire sequence of steps will be repeated periodically, to take into account new information (new radar measurements, changes in the flight plan, changes in the wind conditions, etc.) An obvious choice for when to repeat the conflict detection computation is whenever a new radar measurement comes in (typically every 12 seconds). Because the algorithm may be computationally demanding, however, we may want to repeat the computation more infrequently (e.g. every minute).

Step 0: Initial Data. For each iteration of the conflict detection algorithm we assume that we are given a *prediction horizon*, $T \in \mathbb{R}_+$, the *current position*, $P(0) \in \mathbb{R}^2$, of each aircraft, the *flight plan* $\{(O_i, v_i)\}_{i=0}^N$ for each aircraft, and the *nominal wind profile* $W : \mathbb{R}_+ \times \mathbb{R}^2 \to \mathbb{R}^2$. $P(0)$ is assumed to be a normally distributed random variable, with mean $\overline{P}_0 \in \mathbb{R}^2$ and covariance matrix $Q_0 \in \mathbb{R}^{2\times 2}$. The interpretation is that $\overline{P}_0 \in \mathbb{R}^2$ is the radar measurement, given in a global coordinate frame, and Q_0 reflects our uncertainty about this measurement. We assume that at the current time the aircraft is on its way from way point O_0 to O_1; way points further in the past are discarded.

Step 1: Nominal Trajectory. Simulate the nonlinear hybrid system starting from the discrete state (O_0, ϕ_0, v_0) and with the stochastic perturbation term $N(t, P(t))$ set equal to 0. The outcome of the simulation a sequence of nominal positions, $\{\overline{P}_t\}$, $\overline{P}_t \in \mathbb{R}^2$ for $t = 0, \Delta, 2\Delta, \ldots$. The simulation also generates a function $WP : \{\overline{P}_{k\Delta}\}_{k=0}^m \to \{(O_i, v_i, \phi_i)\}_{i=0}^N$. The interpretation is that if $WP(\overline{P}_{k\Delta}) = (O_i, v_i, \phi_i)$, then at time $k\Delta$ in the future the aircraft will be on its way from way point O_i to way point O_{i+1}.

Step 2: Time Varying Coordinate Change. Suppose that at time $t = k\Delta$, $WP(\overline{P}_t) = (O_i, v_i, \phi_i)$. We define

$$R_t = \begin{bmatrix} \cos\phi_i & \sin\phi_i \\ -\sin\phi_i & \cos\phi_i \end{bmatrix}, \quad \begin{bmatrix} w_{\phi_i}(t) \\ w_{\phi_i^\perp}(t) \end{bmatrix} = R_t W(t, \overline{P}_t),$$

$$\theta_t = -\sin^{-1}\left(\frac{w_{\phi_i^\perp}(t)}{v_i}\right), \quad s_t = -\frac{k_2}{k_3}\theta_t.$$

The coordinate change to linearising coordinates can now be written as

$$\begin{bmatrix} X \\ \delta\theta \\ \delta s \end{bmatrix} = \begin{bmatrix} R_t & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P \\ \psi \\ s \end{bmatrix} - \begin{bmatrix} R_t O_i \\ \phi_i + \theta_t \\ s_t \end{bmatrix}, \quad (5)$$

or, in other words,

$$Y = T_t \begin{bmatrix} P \\ \psi \\ s \end{bmatrix} + S_t, \quad (6)$$

where we set $Y = [X, \delta\theta, \delta s]^T$, and T_t and S_t are the matrices that appear in equation (5).

Step 3: Linear Time Varying Model. A somewhat tedious computation reveals that the evolution of the system from time t to $t + \Delta$ in linearising coordinates Y can be approximated by

$$Y(t + \Delta) = A_t Y(t) + B_t + C_t N(t, \overline{P}_t), \quad (7)$$

for appropriate choices of A_t, B_t and C_t that depend on the time varying coordinate change of Step 2, the FMS gains and the sampling interval Δ. Y is a discrete time stochastic process which evolves from time t according to the linear time-varying equation (7), until the switching time t_{i+1} is reached. At time t_{i+1}, Y is reset, and it then evolves according to equation (7) with appropriately redefined matrices A_t, B_t, C_t.

Step 4: Pairwise Detection. Select a pair of aircraft moving in the same region of the airspace. The remaining steps will be repeated until all pairs of aircraft have been tested.

Let $Y(t)$ and $Y'(t)$ denote the state of the two aircraft at time t in the respective linearised coordinates. The evolution of the two aircraft system can be approximated by a 8×8, discrete time, linear, time varying, stochastic system

$$\begin{bmatrix} Y \\ Y' \end{bmatrix}(t + \Delta) = \begin{bmatrix} A_t & 0 \\ 0 & A'_t \end{bmatrix} \begin{bmatrix} Y \\ Y' \end{bmatrix}(t) + \begin{bmatrix} B_t \\ B'_t \end{bmatrix} + \begin{bmatrix} C_t & 0 \\ 0 & C'_t \end{bmatrix} \begin{bmatrix} N(t, \overline{P}_t) \\ N(t, \overline{P}'_t) \end{bmatrix}, \quad (8)$$

until one of the aircraft turns to a different flight plan segment. For ease of explanation we refer to the case when no aircraft turns in the considered time interval $[0, t]$. Taking into account the occurrence of a turning event will only make the expressions derived below more complex.

We treat $(Y(t), Y'(t))$ as a Gaussian random variable with mean $(m(t), m'(t)) \in \mathbb{R}^8$ and covariance matrix $Q(t) \in \mathbb{R}^{8\times 8}$. This is the case if all the random variables $N(k, \overline{P}_k), N(k, \overline{P}'_k)$, $k = 0, \ldots, t - \Delta$, and the initial state $(Y(0), Y'(0))$ are jointly Gaussian. We assume that $N(k, P)$ and $(Y(0), Y'(0))$ are uncorrelated for all $k \geq 0$ and $P \in \mathbb{R}^2$, and that $Y(0)$ is normally distributed with mean $m(0)$ and covariance matrix $Q(0)$. The mean of $(Y(t), Y'(t))$ can then be computed recursively by

$$\begin{bmatrix} m \\ m' \end{bmatrix}(t + \Delta) = \begin{bmatrix} A_t & 0 \\ 0 & A'_t \end{bmatrix} \begin{bmatrix} m \\ m' \end{bmatrix}(t) + \begin{bmatrix} B_t \\ B'_t \end{bmatrix},$$

where we use the fact that $N(k, P)$ is assumed to be zero mean. The covariance matrix can also be computed recursively (using the fact that $N(k, P)$ is stationary, uncorrelated with $(Y(0), Y(0)')$, and isotropic), but the equations are rather tedious and are omitted.

Step 5: Separation and Overlap Probability. The separation of the two aircraft at time t is $D(t) = P(t) - P'(t)$. The overlap probability at time t is the probability that the random variable $D(t)$ falls in a disc of radius h centred at the origin, where h is the minimum allowed horizontal separation. Using equation (6), it is easy to show that $D(t)$ is an affine function of $Y(t)$ and $Y'(t)$. This implies that the aircraft separation $D(t)$ at time t is a Gaussian random variable taking values in $\mathbb{R}^2$, whose mean and covariance matrix can be computed using the formulas in Step 4. Therefore the overlap probability can be easily approximated either numerically, or using randomised extractions.

5 Concluding Remarks

One problem that needs to be addressed to ensure that the proposed model is realistic is stability analysis. The model of (4) is too complicated to analyse its stability "at one go", since it involves the interaction of nonlinear continuous dynamics, discrete dynamics, and stochastic terms. None of the methods currently in the literature are capable of dealing with this type of system. We propose to develop a method for studying the stability of such stochastic hybrid systems. A good starting point are methods developed for switched systems [9]. Most such methods deal with deterministic continuous time systems and will have to be adapted to be applied to our stochastic discrete time model.

The proposed model contains a number of "free" parameters (the FMS gains, the variances of the wind, etc.) whose values need to be chosen to improve the predictions of the model. To make the system realistic, the exact parameter values should be chosen to match the observed behaviour of real aircraft. This can be done through a process of *system identification*. When we try to pose the problem of selecting parameter values in a system identification framework we immediately run against the problem of *time scales*. FMS control takes place at fairly high frequency (of the order of one sample every second, i.e. 1Hz). Ideally the data used for identification should be sampled at the same frequency. Unfortunately, because the available data is obtained through radar, it is likely to be much more infrequent: less than 0.1Hz if we assume a standard radar sampling time of 12 seconds. In real life data collection experiments, the data may be even more sparse: roughly 1 sample every minute, or 0.017Hz. System identification methods designed specifically to deal with missing data [10] are unlikely to work in this case, since they are designed for situations where the missing samples are sparse (in our case possibly 59 out of 60 samples are missing).

In the long run, one would hope to be able to use models like the one developed here not only to warn air traffic controllers about potential problems, but also to provide suggestions on how to resolve them. As a first step in this direction we are currently investigating how the results of our algorithms can be coupled with the framework of "no-go" zones used by HIPS [11].

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