AN OBSERVER FOR SWITCHING NONLINEAR SYSTEMS WITH MODE DEPENDENT TIME DELAYS

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ABSTRACT: The problem of state reconstruction from input and output measurements for a class of switching nonlinear systems with mode dependent time delays is studied in this paper and a state observer which gives exponential observation error decay is proposed. The mode is supposed to be known. Moreover it is supposed that there exists a minimum time between two consecutive switches (dwell time). The proposed observer is a switching nonlinear time delay system of the neutral type. Simulations are reported that show the effectiveness of the proposed observation algorithm.

Keywords: Switching Nonlinear Systems, Time Delay Systems, State Observers, Infinite Dimensional Systems.

1. Introduction

In [3] an observer is proposed for a class of nonlinear time delay systems, which gives exponential observation error decay. The same approach followed in paper [2] is adopted in [3] for delay systems, and a Luenberger-like observer is obtained which works provided the following main hypothesis is satisfied: the output (in a finite dimensional set) and some of its derivatives are related with the system variables by means of an asymptotically stable continuous time difference equation. In [4] an observer is proposed for switching nonlinear (delayless) systems, based on [2]. This paper is an extension of the works in [3] and [4]. Here an asymptotic observer is built up for a class of switching nonlinear systems with mode dependent time delays. As in [4] for the delayless case, here the mode is supposed to be known and it is supposed that there exists a minimum time between two consecutive switches (dwell time). To my knowledge, no paper concerning the observer problem for switching nonlinear time delay systems has been published in literature. The observer here proposed is a switching nonlinear delay differential equation of the neutral type. The observer error goes to zero with exponential decay rate. The observer reproduces exactly the system equation when initialized by the true initial state. As in [3], the calculation of the output feedback gain is straightforward. As in [3], it is supposed that the output belongs to a finite dimensional set, and the observer is proved to converge provided the same above hypothesis is satisfied as far as the relation between the output and the system variables is concerned. The interested reader can refer to [1] for a general survey on the topic of switching systems with delays. In particular, the section 8.4 in [1] deals with switching linear systems with (known) mode dependent time delays.

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2. Preliminaries

The system under investigation is described by the following equation

$$\dot{x}(t) = f(\sigma(t), x(t), x(t-\Delta(\sigma(t)))) + g(\sigma(t), x(t), x(t-\Delta(\sigma(t))))u(t), \qquad (1)$$

$$y(t) = h(\sigma(t), x(t)), \qquad t \geq 0, \qquad (2)$$

where $x(t) \in I\!R^n$, $u(t) \in I\!R$ and $y(t) \in I\!R$, $\sigma : [0,\infty) \mapsto \mathcal{S}$ is the piecewise constant function describing the system mode, $\mathcal{S} = \{1, 2, \ldots, m\}$, with m a positive integer, is the mode state space, for each $i \in \mathcal{S}$ the vector functions $f(i,*,*)$ and $g(i,*,*)$ are C^∞ with respect to both arguments $*$, and the scalar function $h(i,*)$ is C^∞ with respect to the argument $*$, $\Delta(i) > 0$, $i =$

$1, 2, \ldots, m$, are the mode dependent time delays. The model description is completed by the function $x(\tau)$, $\tau \in [-\Delta_{max}, 0]$, which represents the initial state, where $\Delta_{max} = \sup_{i=1,2,\ldots,m}\{\Delta(i)\}$. As usual, such function is supposed to be in $W^{1,2}([-\Delta_{max}, 0]; I\!R^n)$, the space of absolutely continuous functions with square integrable derivative. Let the symbol $\mathcal{X}_{i,j}$, with i, j non-negative integers, $0 \le i \le j$, denote the composed vector

$$\mathcal{X}_{i,j} = \begin{bmatrix} \chi_i \\ \chi_{i+1} \\ \vdots \\ \chi_j \end{bmatrix} \in I\!R^{(j-i+1)n} \tag{3}$$

where χ_i, i non negative integer, are vectors in $I\!R^n$.

Definition 1. *The switching nonlinear time delay system (1), (2) is said to have observation delay relative degree r in an open set $\Omega_r \in I\!R^{n(r+1)}$ if, defining for each element $s = (s_1, s_2, \ldots, s_r) \in \mathcal{S}^r$,*

$$F(s, \mathcal{X}_{0,r}) = \begin{bmatrix} f(s_1, \chi_0, \chi_1) \\ f(s_2, \chi_1, \chi_2) \\ \vdots \\ f(s_r, \chi_{r-1}, \chi_r) \\ 0_{n\times 1} \end{bmatrix},$$
$$H(s, \mathcal{X}_{0,r}) = h(s_1, \chi_0),$$
$$G(s, \mathcal{X}_{0,r}) = \begin{bmatrix} \operatorname{diag}\{g(s_1, \chi_0, \chi_1), \ldots, g(s_r, \chi_{r-1}, \chi_r)\} \\ 0_{n\times r} \end{bmatrix} \tag{4}$$

the following conditions are verified $\forall \mathcal{X}_{0,r} \in \Omega_r$, $\forall s \in \mathcal{S}^r$,

$$L_G L_F^k H(s, \mathcal{X}_{0,r}) = 0, \qquad k = 0, 1, \ldots, r-2, \tag{5}$$

where

$$\begin{aligned} L_F^0 H(s, \mathcal{X}_{0,r}) &= H(s, \mathcal{X}_{0,r}), \\ L_F^k H(s, \mathcal{X}_{0,r}) &= \left(\frac{\partial}{\partial \mathcal{X}_{0,r}} L_F^{k-1} H\right) F(s, \mathcal{X}_{0,r}), \\ & k \le r \\ L_G L_F^k H(s, \mathcal{X}_{0,r}) &= \left(\frac{\partial}{\partial \mathcal{X}_{0,r}} L_F^{k} H\right) G(s, \mathcal{X}_{0,r}), \\ & k \le r-1 \end{aligned} \tag{6}$$

If $\Omega_r = I\!R^{n(r+1)}$, the system is said to have uniform observation delay relative degree r. ■

For systems having observation delay relative degree equal to r in Ω_r it is

$$\begin{aligned} y^{(k)}(t) &= L_F^k H(J(t), X(J(t), t)), \ k = 0, 1, \ldots r-1 \\ y^{(r)}(t) &= L_F^r H(J(t), X(J(t), t)) + \\ & \quad L_G L_F^{r-1} H(J(t), X(J(t), t)) U(J(t), t), \end{aligned} \tag{7}$$

where:

$J(t) \in \mathcal{S}^r$ is given by

$$J(t) = \begin{bmatrix} j_1(t) \\ j_2(t) \\ \vdots \\ j_r(t) \end{bmatrix} = \begin{bmatrix} \sigma(t) \\ \sigma(t - \Delta(\sigma(t))) \\ \sigma(t - \Delta(\sigma(t)) - \Delta(\sigma(t - \Delta(\sigma(t))))) \\ \vdots \end{bmatrix}, \tag{8}$$

$U(J(t), t) \in I\!R^r$ and $X(J(t), t) \in I\!R^{n(r+1)}$ are given by

$$U(J(t), t) = \begin{bmatrix} u(t) \\ u(t - \Delta(j_1(t))) \\ u(t - \Delta(j_1(t)) - \Delta(j_2(t))) \\ \vdots \\ u\left(t - \sum_{i=1}^{r-1} \Delta(j_i(t))\right) \end{bmatrix}, \tag{9}$$

$$X(J(t), t) = \begin{bmatrix} x(t) \\ x(t - \Delta(j_1(t))) \\ x(t - \Delta(j_1(t)) - \Delta(j_2(t))) \\ \cdots \\ x\left(t - \sum_{i=1}^{r} \Delta(j_i(t))\right) \end{bmatrix}. \tag{10}$$

The following function $\phi : \mathcal{S}^r \times \Omega_r \to I\!R^n$ can be defined for systems having observation delay relative degree equal to r in Ω_r:

$$\phi(s, \chi_{0,r}) = \phi(s, \chi_0, \chi_1, \ldots, \chi_r) = \begin{bmatrix} H(s, \chi_{0,r}) \\ L_F H(s, \chi_{0,r}) \\ \vdots \\ L_F^{r-1} H(s, \chi_{0,r}) \end{bmatrix} \tag{11}$$

Remark 2. The value of the function ϕ does not depend on the last argument, χ_r. ■

Again, for systems having observation delay relative degree equal to r in Ω_r, by defining $z(t)$ as the vector of the output and its $(r-1)$ time derivatives, that is

$$z(t) = \begin{bmatrix} y(t) \\ y^{(1)}(t) \\ \vdots \\ y^{(r-1)}(t) \end{bmatrix}, \quad (12)$$

the following equality holds in Ω_r:

$$\begin{aligned} z(t) =& \phi(J(t), X(J(t),t)) = \\ & \phi(J(t), x(t), x(t-\Delta(j_1(t))), \ldots, \\ & x(t - \sum_{i=1}^{r} \Delta(j_i(t)))) \end{aligned} \quad (13)$$

Take into account that in general (13) holds for $t \geq (r-1)\Delta_{max}$.

¿From here on the following hypotheses are supposed satisfied:

Hp_0) *the system (1), (2) has uniform observation delay relative degree r equal to n (the length of the vector $x(t)$);*

Hp_1) *there exists a function $\phi^{-1} : \mathcal{S}^n \times I\!R^{n(n+1)} \to I\!R^n$ such that, for all*

$$\begin{aligned} & (s, z, \chi_0, \chi_1, \chi_2, \ldots, \chi_n) \in \mathcal{S}^n \times I\!R^n \times I\!R^{n(n+1)}, \\ & z = \phi\big(s, \phi^{-1}\big(s, z, \chi_1, \cdots, \chi_n\big), \chi_1, \cdots, \chi_n\big), \quad (14) \\ & \chi_0 = \phi^{-1}\big(s, \phi\big(s, \chi_0, \chi_1, \cdots, \chi_n\big), \chi_1, \cdots, \chi_n\big); \quad (15) \end{aligned}$$

moreover, $\forall\ s \in \mathcal{S}^n$, the function $\phi^{-1}(s, *)$ is continuous and admits continuous partial derivatives of any order in $I\!R^{n(n+1)}$;

Hp_2) *at the current time t, $\sigma(\tau)$ is known, $0 \leq \tau \leq t$; there exists a positive real τ_D (dwell time), such that the time between two consecutive switches is not less than τ_D.*

By the hypothesis Hp_0, the following equation can be derived from the dynamics of the system (1), (2) :

$$\begin{aligned} \dot{z}(t) = Az(t) + B\Big(& L_F^n H(J(t), x(t), x(t-\Delta(j_1(t))), \\ & \cdots, x(t - \sum_{i=1}^{n} \Delta(j_i(t))))+ \\ & L_G L_F^{n-1} H(J(t), x(t), x(t-\Delta(j_1(t))), \cdots, \\ & x(t - \sum_{i=1}^{n} \Delta(j_i(t))))U(J(t),t)\Big), \\ y(t) = Cz(t), & \qquad t \geq (n-1)\Delta_{max}, \end{aligned} \quad (16)$$

where the triple A, B, C is a Brunowsky triple, i.e.

$$A = \begin{bmatrix} 0_{(n-1)\times 1} & I_{n-1} \\ 0 & 0_{1\times(n-1)} \end{bmatrix}, \quad B = \begin{bmatrix} 0_{(n-1)\times 1} \\ 1 \end{bmatrix}, \\ C = [\ 1 \quad 0_{1\times(n-1)}\ ]. \quad (17)$$

The pair A, C is observable.

¿From the hypotheses Hp_0 and Hp_1 it follows that the matrix function

$$Q : \mathcal{S}^n \times I\!R^{n(n+1)} \mapsto I\!R^{n\times n} \quad (18)$$

defined as

$$Q(s, \chi_0, \chi_1, \cdots, \chi_n) = \frac{\partial\phi(s, \chi_0, \chi_1, \cdots, \chi_n)}{\partial\chi_0} \quad (19)$$

is nonsingular for all $(s, \chi_0, \chi_1, \cdots, \chi_n) \in \mathcal{S}^n \times I\!R^{n(n+1)}$.

We will refer to matrix Q as the delay drift-observability matrix associated to system (1), (2).

3. The Observer for Switching Delay Systems

The proposed observer for switching nonlinear time delay systems (1) (2), that satisfy the hypotheses Hp_0, Hp_1, Hp_2, is the following

$$\begin{aligned} \dot{\hat{x}}(t) &= f(\sigma(t), \hat{x}(t), \hat{x}(t-\Delta(\sigma(t)))+ \\ & \quad g(\sigma(t), \hat{x}(t), \hat{x}(t-\Delta(\sigma(t)))u(t) + w(t), \ t \geq 0, \\ w(t) &= Q^{-1}(J(t), \hat{X}(J(t),t))K\left(y(t) - h(\sigma(t), \hat{x}(t))\right) \\ & \quad - Q^{-1}(J(t), \hat{X}(J(t),t)) \cdot \\ & \quad \sum_{i=1}^{n-1} \frac{\partial\phi(J(t), \hat{X}(J(t),t))}{\partial\hat{x}(t - \sum_{l=1}^{i} \Delta(j_l(t)))} w(t - \sum_{l=1}^{i} \Delta(j_l(t))) \end{aligned} \quad (20)$$

with initial conditions

$$\hat{x}(\tau) = \xi(\tau), \ w(\tau) = \eta(\tau), \ \tau \in [-(n-1)\Delta_{max}, 0] \\ \xi, \eta \in W^{1,2}([-(n-1)\Delta_{max}, 0]; I\!R^n)$$

Here

$$\hat{X}(J(t),t) = [\,\hat{x}(t), \hat{x}(t-\Delta(j_1(t)), \ldots, \hat{x}(t-\Delta(j_n(t)))\,]^T,$$

and Q^{-1} is the inverse of the delay drift-observability matrix Q. When the term $J(t)$ involves the term $\sigma(\tau), \tau < 0$, (which is not given), then in the observer equation it is set $\sigma(\tau) = \sigma(0)$. The gain vector $K \in I\!R^n$ is chosen such to assign stable eigenvalues to the matrix $A - KC$. The part in $[-\Delta_{max}, 0]$ of the function ξ that initializes the observer represents the priori knowledge on the system state. If the exact $x(t)$ is known for $t \in [-\Delta_{max}, 0]$, the observer initialization is made as follows

$$\hat{x}(\tau) = \bar{\xi}(\tau), \; w(\tau) = 0, \; \tau \in [-(n-1)\Delta_{max}, 0] \tag{21}$$

where $\bar{\xi}$ is any function in $W^{1,2}([-(n-1)\Delta_{max}, 0]; I\!R^n)$ such that $\bar{\xi}(\tau) = x(\tau), \tau \in [-\Delta_{max}, 0]$. In this case the feedback terms in (20) are identically zero and for all t it is $\hat{x}(t) = x(t)$.

Note that if for a given $\bar{t}$ it is $\hat{x}(\tau) = x(\tau)$ for $\tau \in [\bar{t}-(n-1)\Delta_{max}, \bar{t}]$, then $\hat{x}(t) = x(t)$ for all $t > \bar{t}$ (it is easy to check that, in this case, $w(\tau) = 0$ and $y(\tau) = h(\sigma(\tau), \hat{x}(\tau))$ for $\tau \in [\bar{t} - (n-1)\Delta_{max}, \bar{t}]$, so that for $t \geq \bar{t}$ the feedback terms in (20) are identically zero).

For the observer (20) the following convergence theorem can be given.

Theorem 3. *Let the system* (1), (2) *satisfy the hypotheses* Hp_0, Hp_1, Hp_2. *Assume there exists* u_M *such that* $|u(t)| \leq u_M$ $\forall t \geq 0$. *Moreover assume the following Lipschitz hypotheses:*

H1: there exists a positive constant α_0 *such that, for any given*

$$[\, z \quad \chi_1 \quad \ldots \quad \chi_n \,]^T, [\, \hat{z} \quad \hat{\chi}_1 \quad \ldots \quad \hat{\chi}_n \,]^T \in I\!R^{n(n+1)},$$

$$\sup_{\|U\| \leq n u_M, s \in \mathcal{S}^n} \|L_F^n H(s, \phi^{-1}(s, z, \chi_1, \ldots, \chi_n), \chi_1, \ldots, \chi_n) - L_F^n H(s, \phi^{-1}(\hat{z}, \hat{\chi}_1, \ldots, \hat{\chi}_n), \hat{\chi}_1, \ldots, \hat{\chi}_n) + L_G L_F^{n-1} H(s, \phi^{-1}(s, z, \chi_1, \ldots, \chi_n), \chi_1, \ldots, \chi_n)U - L_G L_F^{n-1} H(s, \phi^{-1}(s, z, \chi_1, \ldots, \chi_n), \chi_1, \ldots, \chi_n)U\| \leq \alpha_0 \left\| \begin{matrix} z - \hat{z} \\ \chi_1 - \hat{\chi}_1 \\ \vdots \\ \chi_n - \hat{\chi}_n \end{matrix} \right\| \tag{22}$$

H2: there exist positive constants α *and* γ,

$$\gamma < \frac{1}{(n-1)}, \tag{23}$$

such that, for any given

$$[\, z \quad \chi_1 \quad \ldots \quad \chi_n \,]^T, [\, \hat{z} \quad \hat{\chi}_1 \quad \ldots \quad \hat{\chi}_n \,]^T \in I\!R^{n(n+1)}, \tag{24}$$

$$\sup_{s \in \mathcal{S}^n} \|\phi^{-1}(s, z, \chi_1, \ldots, \chi_n) - \phi^{-1}(s, \hat{z}, \hat{\chi}_1, \ldots, \hat{\chi}_n)\| \leq \alpha \|z - \hat{z}\| + \gamma \left\| \begin{matrix} \chi_1 - \hat{\chi}_1 \\ \vdots \\ \chi_n - \hat{\chi}_n \end{matrix} \right\| \tag{25}$$

Then, there exists a gain vector $K \in I\!R^n$ *to be put in the observer* (20) *such that*

$$\lim_{t \to \infty} \|x(t) - \hat{x}(t)\| = 0. \tag{26}$$

Moreover, such convergence to zero is exponential, with exponential decay rate arbitrarily fixed by the choice of K *among the negative reals* c *such that*

$$1 - \gamma \sum_{i=1}^{n-1} e^{-ci\Delta_{max}} > 0.$$

■

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