A State Observer for a Class of Nonlinear Systems with Multiple Discrete and Distributed Time Delays

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Abstract

This paper considers the state observer problem for a class of nonlinear systems, which present multiple non commensurate time delays as well as distributed delay terms. The proposed algorithm is an extension of the observer for nonlinear delayless systems proposed in 1993 by Ciccarella, Dalla Mora, Germani. It is proved that a suitable gain can be easily chosen such that the observation error goes to zero exponentially, with arbitrarily fixed decay rate. The algorithm presented here is a basis for future developments of observers for hybrid systems with multiple discrete and distributed mode dependent time delays.

1. Introduction

The only paper available in the literature on the topic of the observer problem for nonlinear systems with delays in the state is, to our knowledge, [9]. There an asymptotic state observer is built up for a class of nonlinear systems with one only discrete delay, and no distributed delay terms. In this paper the observer problem for another class of nonlinear time delay systems is considered. These systems can have multiple noncommensurate and distributed time delays. The output and its $n-1$ time derivatives are related with the system variables by a diffeomorphism, as in [2]. Therefore no continuous time difference equation dynamics are involved. The class of systems considered in this paper is not contained in the class of systems considered in [9]. The class of systems considered in this paper contains the class of systems (one discrete delay) studied in [9], only when the output of the systems in [9] and its $n-1$ time derivatives depend on the present value of the system variables, and not on their past ones. The observer proposed in this paper has the property that the observation error goes exponentially to zero, with any prescribed decay rate. Moreover the initial conditions of both the system and the observer are allowed to be discontinuous. The gain is very easy to be calculated, as the one for nonlinear delayless systems proposed in [2]. The importance of the class of systems considered in this paper is due to their use in the literature to model practical problems such as biological phenomena. For instance, the glucose-insuline (see [5,14]) and the prey-predator Lotka-Volterra (see [15]) models are in that class. Moreover, the algorithm presented here is a basis for future developments of observers for hybrid systems with multiple discrete and distributed mode dependent time delays (see [10,21]).

This work is supported by the European Project IST-2001-32460 HYBRIDGE, and by the Italian Project MIUR PRIN 2003.

2. Preliminaries

Consider the following system

$$\dot{x}(t) = f(x(t)) + g(x(t)) \cdot p(x_t, u(t)), \tag{2.1}$$

$$y(t) = h(x(t)), \qquad t \geq 0, \tag{2.2}$$

with initial conditions $x_0 \in PC([-\Delta, 0]; I\!R^n)$ (here PC denotes the set of functions which are bounded, and are continuous except in a finite number of points), where:

$\Delta > 0$ is the maximum delay, $x(t) \in I\!R^n$, $u(t) \in I\!R$, $y(t) \in I\!R$, $f, g : I\!R^n \mapsto I\!R^n$ are C^∞ vector functions, $h : I\!R^n \mapsto I\!R$ is a C^∞ scalar function, p is a functional from $PC([-\Delta, 0]; I\!R^n) \times I\!R$ to $I\!R$, $x_t \in PC([-\Delta, 0]; I\!R^n)$ is given by $x_t(\tau) = x(t + \tau)$.

From here on we suppose that the following hypothesis is satisfied by the system (2.1)(2.2):

H_p) the triple (f, g, h) is such that, $\forall\ \chi_0 \in I\!R^n$, $l_g l_f^k h(\chi_0) = 0$, $k = 0, 1, \ldots, n-2$; the function $\phi : I\!R^n \mapsto I\!R^n$, defined by $\phi(\chi_0) = [\, h(\chi_0) \quad l_f h(\chi_0) \quad \ldots \quad l_f^{n-1} h(\chi_0)\,]^T$, $\chi_0 \in I\!R^n$, is a diffeomorphism in $I\!R^n$.

Let $y^{(j)}(t)$ be the $j-th$ time derivative of the measured output $y(t)$, $j = 1, 2, \ldots, n$. Let

$$z^T(t) = [\, y(t) \quad y^{(1)}(t) \quad \ldots \quad y^{(n-1)}(t)\,]^T, \ t \geq 0, \qquad z(\tau) = \phi(x_0(\tau)), \tau \in [-\Delta, 0] \tag{2.3}$$

The system (2.1)(2.2) can be rewritten in normal form as

$$\begin{aligned} \dot{z}(t) &= Az(t) + B(l_f^n h(\phi^{-1}(z(t))) + l_g l_f^{n-1} h(\phi^{-1}(z(t))) \cdot \\ &\qquad p(\Phi^{-1}(z_t), u(t)) \\ z(\tau) &= \phi(x_0(\tau)), \tau \in [-\Delta, 0] \end{aligned} \tag{2.4}$$

where: A, B have the Brunowsky canonical form (see [13], pp. 153, 231); ϕ^{-1} is the inverse function of ϕ; $\Phi : PC([-\Delta, 0]; I\!R^n) \mapsto PC([-\Delta, 0]; I\!R^n)$ is given by

$$\Phi(\psi)(\tau) = \phi(\psi(\tau)), \qquad \psi \in PC([-\Delta, 0]; I\!R^n), \qquad \tau \in [-\Delta, 0]; \tag{2.5}$$

$\Phi^{-1} : PC([-\Delta, 0]; I\!R^n) \mapsto PC([-\Delta, 0]; I\!R^n)$ is given by

$$\Phi^{-1}(\psi)(\tau) = \phi^{-1}(\psi(\tau)), \qquad \psi \in PC([-\Delta, 0]; I\!R^n), \qquad \tau \in [-\Delta, 0]. \tag{2.6}$$

By the hypothesis H_p it follows that the matrix function defined by

$$Q(\chi_0) = \frac{\partial \phi(\chi_0)}{\partial \chi_0}, \qquad \chi_0 \in I\!R^n \tag{2.7}$$

is invertible for any $\chi_0 \in I\!R^n$.

3. The Proposed Observer

The proposed observer for nonlinear delay systems (2.1)(2.2) is the following

$$\dot{\hat{x}}(t) = f(\hat{x}(t)) + g(\hat{x}(t)) \cdot p(\hat{x}_t, u(t)) + Q^{-1}(\hat{x}(t)) K(y(t) - h(\hat{x}(t)), \qquad t \geq 0, \tag{3.1}$$

with initial conditions $\hat{x}_0 \in PC([-\Delta, 0] \mapsto I\!R^n$. As usual, $\hat{x}_t(\tau) = \hat{x}(t+\tau)$, $\tau \in [-\Delta, 0]$.

The gain vector $K \in I\!R^n$ is chosen such to assign the eigenvalues of the matrix $A - KC$ in the left half complex plane.

Note that if for a given $\bar{t}$ it is $\hat{x}(\tau) = x(\tau)$ for $\tau \in [\bar{t} - \Delta, \bar{t}]$, it follows that $\hat{x}(t) = x(t)$ for all $t > \bar{t}$.

Lemma 3.1. *The dynamics of the observer (3.1) can be rewritten as*

$$\begin{aligned}
\dot{\hat{z}}(t) &= A\hat{z}(t) + B(l_f^n h(\phi^{-1}(\hat{z}(t))) + l_g l_f^{n-1} h(\phi^{-1}(\hat{z}(t)))\cdot \\
&\qquad p(\Phi^{-1}(\hat{z}_t), u(t))) + K(y(t) - C\hat{z}(t)), \\
\hat{x}(t) &= \phi^{-1}(\hat{z}(t)), \\
\hat{z}(\tau) &= \phi(\hat{x}_0(\tau)), \tau \in [-\Delta, 0]
\end{aligned} \tag{3.2}$$

where A, B, C have the Brunowsky canonical form, and $\hat{z}_t(\tau) = \hat{z}(t+\tau)$, $\tau \in [-\Delta, 0]$. ■

Lemma 3.2. *Let ω_1, ω_2 be scalar non negative functions defined on $[-\Delta, \infty)$, such that:*

1) *they are bounded in $[-\Delta, 0]$, and they are continuous in $[-\Delta, 0]$ except in a finite number of points;*
2) *they are continuous in $[0, \infty)$;*
3) *$\omega_1(\tau) < \omega_2(\tau)$, $\tau \in [-\Delta, 0]$.*

Let η_1, η_2 be scalar non negative continuous functions defined on $[0, \infty)$, with $\eta_1(t) < \eta_2(t)$, $t \geq 0$. Let W be a non negative scalar continuous function defined on $[0, \infty) \times [0, \infty)$. Let, for $t \geq 0$,

$$\omega_1(t) \leq \eta_1(t) + \int_0^t W(t, \tau) \sup_{\theta \in [-\Delta, 0]} \omega_1(\tau + \theta) d\tau \tag{3.3}$$

$$\omega_2(t) = \eta_2(t) + \int_0^t W(t, \tau) \sup_{\theta \in [-\Delta, 0]} \omega_2(\tau + \theta) d\tau \tag{3.4}$$

Then, $\omega_1(t) < \omega_2(t)$, $t \geq 0$.

Lemma 3.3. (Bellman-Gronwall) *Let $\omega : [0, \infty) \mapsto I\!R^+$ be a continuous function which:*

1_ω) *is left-hand differentiable in $(0, \infty)$ and right-hand differentiable in $[0, \infty)$;*
2_ω) *in any bounded interval contained in $(0, \infty)$, admits a finite number of points where the left-hand derivative is different from the right-hand derivative.*

Let $g : [-\Delta, \infty) \mapsto I\!R^+$ be a function which:

1_g) *is bounded in any bounded interval;*
2_g) *in any bounded interval is continuous except in a finite number of points.*

Let α, β be non negative reals. Let the following inequality be satisfied for $t \in [0, \infty)$:

$$\omega(t) \leq \alpha + \int_0^t g(\tau) d\tau + \int_0^t \beta \omega(\tau) d\tau \tag{3.5}$$

Then, the following inequality holds

$$\omega(t) \leq e^{\beta t} \alpha + \int_0^t e^{\beta(t-\tau)} g(\tau) d\tau \tag{3.6}$$

■

Theorem 3.4. *Let the system (2.1)(2.2) satisfy the hypothesis H_p. Assume there exists u_M such that $|u(t)| \leq u_M$ $\forall t \geq 0$. Assume the following Lipschitz hypotheses:*
H1) there exist positive constants $\gamma_1, \gamma_2, \gamma_3$ such that, for all $v_1,\ v_2 \in I\!R^n$ and for all $\psi_1, \psi_2 \in PC([-\Delta, 0]; I\!R^n)$,

$$|l_f^n h(\phi^{-1}(v_1)) - l_f^n h(\phi^{-1}(v_2))| \leq \gamma_1 \|v_1 - v_2\|, \tag{3.7}$$

$$\sup_{\|u\| \leq u_M} |l_g l_f^{n-1} h(\phi^{-1}(v_1)) \cdot p(\Phi^{-1}(\psi_1), u) - l_g l_f^{n-1} h(\phi^{-1}(v_2)) \cdot p(\Phi^{-1}(\psi_2), u)| \leq$$
$$\gamma_2 \|v_1 - v_2\| + \gamma_3 \|\psi_1 - \psi_2\|_\infty;$$

H2) there exist positive constants γ_ϕ and $\gamma_{\phi^{-1}}$ such that for all $v_1, v_2 \in I\!R^n$,

$$\begin{aligned} &\|\phi(v_1) - \phi(v_2)\| \leq \gamma_\phi \|v_1 - v_2\|; \\ &\|\phi^{-1}(v_1) - \phi^{-1}(v_2)\| \leq \gamma_{\phi^{-1}} \|v_1 - v_2\|. \end{aligned} \tag{3.8}$$

Then, given any negative real c, there exist a gain K to be put in the observer (3.1), such that

$$\|x(t) - \hat{x}(t)\| \leq \gamma_\phi \gamma_{\phi^{-1}} \|V^{-1}(\lambda)\| \|V(\lambda)\| \|x_0 - \hat{x}_0\|_\infty e^{ct}, \tag{3.9}$$

where λ is the vector of eigenvalues of $A - KC$ and $V(\lambda)$ is the Vandermonde Matrix.

Proof. The proof follows the same lines of the one for the nonlinear observer in [2].

Let $\lambda = [\lambda_1 \quad \lambda_2 \quad \ldots \quad \lambda_n]$ be a $n - pla$ of negative real eigenvalues, with $\lambda_1 > \lambda_2 >, \ldots, > \lambda_n$. Consider the expression (2.4) of system (2.1) and the expression (*3.2*) of the observer.

The dynamics of the error $e_z = z - \hat{z}$ can be written as

$$\begin{aligned} \dot{e}_z(t) = &(A - KC) e_z(t) + B(\\ &l_f^n h(\phi^{-1}(z(t))) + l_g l_f^{n-1} h(\phi^{-1}(z(t))) \cdot p(\Phi^{-1}(z_t), u(t)) - \\ &l_f^n h(\phi^{-1}(\hat{z}(t))) - l_g l_f^{n-1} h(\phi^{-1}(\hat{z}(t))) \cdot p(\Phi^{-1}(\hat{z}_t), u(t))) \end{aligned} \tag{3.10}$$

where the matrix K is such that the matrix $A - KC$ has λ as eigenvalues.

Let $E(t) = V(\lambda) e_z(t)$. By (3.10) and H1, it follows

$$\|E(t)\| \leq e^{\lambda_1 t} \|E(0)\| + \int_0^t e^{\lambda_1 (t-\tau)} \sqrt{n} \gamma \|V^{-1}(\lambda)\| (\|E(\tau)\| + \|E_\tau\|_\infty) d\tau, \tag{3.11}$$

where $\gamma = max(\gamma_1 + \gamma_2, \gamma_3)$. By considering $\psi(t) = e^{-\lambda_1 t} \|E(\tau)\|$ and applying the Belmann-Gronwall Lemma 3.3, it follows

$$\|E(t)\| \leq e^{(\sqrt{n}\gamma \|V^{-1}(\lambda)\| + \lambda_1) t} \|E(0)\| + \int_0^t e^{(\sqrt{n}\gamma \|V^{-1}(\lambda)\| + \lambda_1)(t-\tau)} \sqrt{n} \gamma \|V^{-1}(\lambda)\| \|E_\tau\|_\infty d\tau \tag{3.12}$$

Let c be an arbitrary negative constant.
Let λ such that [2]

$$\sqrt{n}\gamma(1+e^{-c\Delta})\|V^{-1}(\lambda)\|+\lambda_1=c \tag{3.13}$$

Let M be a positive constant such that

$$M>\gamma_\phi\|V(\lambda)\|\|x_0-\hat{x}_0\|_\infty \tag{3.14}$$

Then the function

$$k(t)=Me^{ct}, \qquad t\in[-\Delta,\infty), \tag{3.15}$$

is such that:

$$k(t)=e^{(\sqrt{n}\gamma\|V^{-1}(\lambda)\|+\lambda_1)t}M+\int_0^t e^{(\sqrt{n}\gamma\|V^{-1}(\lambda)\|+\lambda_1)(t-\tau)}\gamma\|V^{-1}(\lambda)\|\|k_\tau|_\infty d\tau \tag{3.16}$$

where $|k_\tau|_\infty=sup_{\theta\in[-\Delta,0]}k(\tau+\theta)=k(\tau-\Delta)$. So, by lemma (3.2) and H_2, it follows $\|x(t)-\hat{x}(t)\|<\gamma_{\phi^{-1}}\|V^{-1}(\lambda)\|k(t)$, which, by arbitrarity of $M>\gamma_\phi\|V(\lambda)\|\|x_0-\hat{x}_0\|_\infty$, proves the theorem. ∎

4. Simulation Results

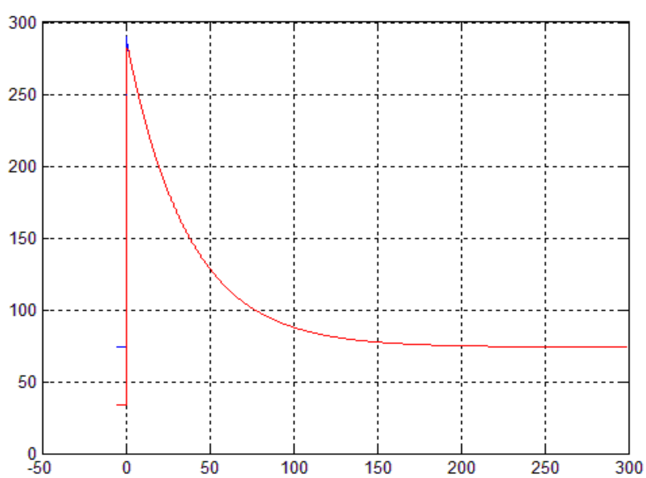

fig.3: true and estimated glucose

Consider the following model of the glucose-insulin system (see [5], pp. 160, [14], pp. 79).

$$\begin{aligned}
&\dot{x}_1(t)=-b_1x_1(t)-b_4x_1(t)x_2(t)+b_7,\\
&\dot{x}_2(t)=-b_2x_2(t)+\frac{b_6}{b_5}\int_{-b_5}^{0}x_2(t+\tau)d\tau,\\
&y(t)=x_1(t),\\
&x_1(\tau)=G_b,\ \tau\in[-b_5,0), \qquad x_1(0)=G_b+b_0,\\
&x_2(\tau)=I_b,\ \tau\in[-b_5,0), \qquad x_2(0)=I_b+b_3b_0,
\end{aligned} \tag{4.1}$$

where x_1 is the glucose, x_2 is the insuline, $b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7$ are parameters, G_b, I_b are initial conditions, which have been taken here equal to the ones proposed in [5], table 1, concerning a 23 years old man. Note that the system presents a distributed delay term and the initial conditions are not continuous in 0. The observer is given by the following system, obtained by (3.1):

$$\dot{\hat{x}}(t) = \begin{bmatrix} \dot{\hat{x}}_1(t) \\ \dot{\hat{x}}_2(t) \end{bmatrix} = \begin{bmatrix} -b_1\hat{x}_1(t) - b_4\hat{x}_1(t)\hat{x}_2(t) + b_7 \\ -b_2\hat{x}_2(t) + \frac{b_6}{b_5}\int_{-b_5}^{0} \hat{x}_2(t+\tau)d\tau \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -b_1 - b_4\hat{x}_2(t) & -b_4\hat{x}_1(t) \end{bmatrix}^{-1} \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} (y(t) - \hat{x}_1(t)), \tag{4.2}$$

with initial conditions $\hat{x}_0 \in PC([-\Delta, 0]), I\!R^2)$. The initial conditions of the observer are taken 40 units lower than the initial conditions of the system.

The vector $K = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}$ in the observer is chosen equal to $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$, by which the matrix $A - KC$ has eigenvalues $(-1, -2)$. In the figures 3 and 4 the true and estimated values of the glucose and of the insuline are reported.

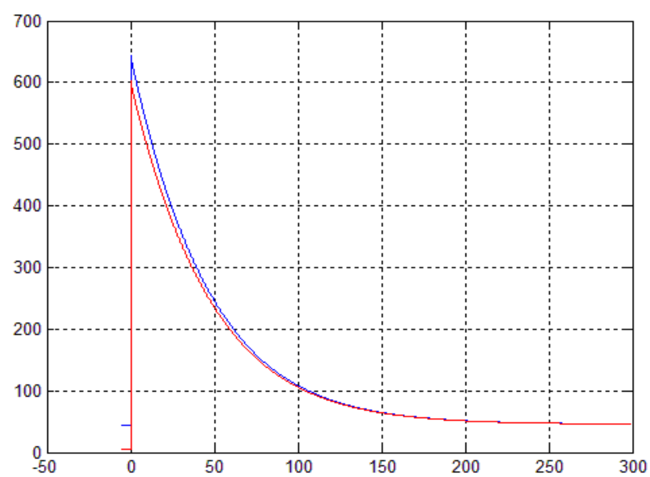

fig.4: true and estimated insuline

For detailed results on the glucose-insuline system and on the problem of the observation of the insuline from the measures of the glucose, which is out of the aims of this theoretical paper, the interested reader can refer to the recent Ph.D. dissertation [6].

References

[1] E. -K. Boukas and Z. -K. Liu, Deterministic and Stochastic Time Delay Systems, *Birkhauser*, 2002.

[2] G. Ciccarella, M. Dalla Mora, A. Germani: "A Luenberger-like observer for non-linear systems", *International Journal of Control*, 1993, vol. 57, no. 3, 537–556.

[3] M. Dalla Mora, A. Germani, C. Manes, Design of State Observers from a Drift-Observability Property, *IEEE Trans. Automat. Contr.*, August 2000.

[4] M. Darouach, "Linear Functional Observers for Systems with Delays in State Variables", *IEEE Transactions on Automatic Control*, Vol. 46, No. 3, March 2001, pp. 491–496.

[5] A. De Gaetano, O. Arino, "Mathematical Modelling of the Intravenous Glucose Tolerance Test", Journal of Mathematical Biology, 40, pp. 136–168, 2000.

[6] D. Di Martino, Modellistica e Identificazione dell'Omeostasi Glicemia - Insulinemia, Ph.D. Dissertation, University of L'Aquila, Italy, 2003.

[7] F. W. Fairman, A. Kumar, Delayless Observers for Systems with Delay, *IEEE Transactions on Automatic Control*, Vol. AC-31, No. 3, March 1986, pp. 258–259.

[8] A. Germani, C. Manes and P. Pepe, Local Asymptotic Stability for Nonlinear State Feedback Delay Systems, *Kybernetika,*, Vol. 36, n. 1, pp. 31-42, 2000

[9] A. Germani, C. Manes and P. Pepe, An Asymptotic State Observer for a Class of Nonlinear Delay Systems, *Kybernetika*, Vol. 37, 2001, N. 4, pp. 459–478.

[10] A. Germani, C. Manes, P. Pepe, "State Observation for Nonlinear Hybrid Automata", WSEAS Transactions on Systems, Vol. 1, N. 2, pp. 210–215, April 2002.

[11] A. Germani, C. Manes, P. Pepe, "A New Approach to State Observation of Nonlinear Systems with Delayed Output", IEEE Transactions on Automatic Control, Vol. 47, No. 1, pp. 96–101, January 2002.

[12] J. K. Hale and S. M. Verduyn Lunel, Introduction to Functional Differential Equations, *Springer Verlag*, 1993

[13] A. Isidori, Nonlinear Control Systems, *Springer-Verlag, Berlin*, 1989.

[14] V. Kolmanovskii and A. Myshkis, Introduction to the Theory and Applications of Functional Differential Equations, *Kluwer Academic Publishers*, 1999.

[15] Y. Kuang, Delay Differential Equations with Applications in Population Dynamics, *Academic Press*, 1993.

[16] M. S. Mahmoud, Robust Control and Filtering for Time-Delay Systems, *Marcel Dekker, New York*, 2000.

[17] S.-I. Niculescu, Delay Effects on Stability, a Robust Control Approach, *Lecture Notes in Control and Information Sciences*, Springer, 2001

[18] T. Oguchi, A. Watanabe and T. Nakamizo, "Input-Output Linearization of Retarded Non-linear Systems by Using an Extension of Lie Derivative", Int. J. Control, Vol. 75, No. 8, 582–590, 2002.

[19] Andrzej W. Olbrot, Observability and Observers for a Class of Linear Systems with Delays, *IEEE Transactions on Automatic Control*, Vol. AC-26, No. 2, April 1981, pp. 513–517.

[20] A. E. Pearson, Y. A. Fiagbedzi, An Observer for Time Lag Systems, *IEEE Transactions on Automatic Control*, Vol. 34, No. 7, July 1989, pp. 775–777.

[21] P. Pepe, "An Observer for Switching Nonlinear Systems with Mode Dependent Time Delays", WSEAS Transactions on Systems, Vol. 3, No. 1, pp. 125-128, January 2004;

[22] Olivier Sename, "New Trends in Design of Observers for Time-Delay Systems", *Kybernetika*, vol. 37, n. 4, pp. 427-458, 2001