Structural discrete state space decompositions for a class of hybrid systems [1]

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Abstract

Stability and observability properties for hybrid systems have been investigated in the literature. However, determining in a computational efficient manner whether hybrid systems possess these important structural properties is still an open problem. Since the difficulty is mainly caused by cycles in the finite state machine part of the system, we offer a number of techniques for eliminating cycles in the structure of the finite state machine so that a significant reduction of the effort required to test stabilizability and detectability can be achieved.

keywords: Linear hybrid systems, switching systems, switched systems, stabilizability, detectability.

1 Introduction

A rich set of results on stability and observability for hybrid systems (see e.g. [4], [8] and references therein for stability properties, [1], [3], [5], [14] for observability) is available in the literature, but the determination of stability conditions that could be efficiently tested is still an open problem. Some sufficient conditions are available but there is no estimate of how conservative these conditions are. The difficulty in obtaining bounds is mainly due to cycles in the finite state machine associated with the hybrid system.

In [7] we proved that checking stabilizability for a switching system is equivalent to check stability of an appropriate autonomous switching system, obtained from the given one with a suitably defined Kalman-like decomposition of the continuous dynamics.

As far as observability is concerned, the condition proposed by the authors in [5] can be easily checked. Detectability conditions are not as easy to check since they depend on the stability of a subsystem of the given system. We believe that simplifying the topological structure of the FSM associated to the hybrid system is crucial to making this computation efficient. In fact, eliminating cycles in the FSM (in some cases this can be done by removing just one discrete state) makes stability trivially verifiable. To the best of our knowledge, the only contribution dealing with structural decomposition of the discrete state space to simplify analysis is reported in [16].

In this paper we offer a number of techniques for simplifying the structure of the FSM along the lines of the structural approach used in [2] and in [6] to solve control problems for hybrid systems with safety constraints.

The paper is organized as follows: in Section 2, the class of linear hybrid systems is defined and the evolution in time of a closed loop controlled system is described. In the first subsection of Section 3, we discuss stabilizability via time-varying state feedback laws, and then, we propose to decompose the discrete state-space to facilitate stability testing. In the second subsection, similar decompositions are applied to simplify the check for detectability.

2 Definitions

We introduce the subclass $\mathcal{H}$ of hybrid systems where the continuous dynamics and the reset functions are linear and the discrete transitions depend either on a disturbance event (switching transitions) or on a discrete input (controlled transitions). This sub-class can be viewed as a particular case of general hybrid systems as defined in [13]: we assume here that the transitions do not depend on the value of the continuous state, or, in other words, that, for any transition, the *guard condition* is the entire continuous state space and, for any discrete state, the *invariance condition* is the con-

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tinuous state space associated to that discrete state. A system in $\mathcal{H}$ is called *linear hybrid system.* The continuous state space associated with each discrete state is characterized by its own dimension that is not necessarily the same for all the discrete states.

Definition 1 *A linear hybrid system $\mathcal{S} \in \mathcal{H}$ is a tuple $(\Xi, \Upsilon, \Theta, \mathbf{S}, S, E, R)$ where:*

- $\Xi = \bigcup_{i \in J} \{q_i\} \times \mathbb{R}^{n_i}$ *is the hybrid state space;* $J = \{1, 2, \ldots, N\}$*;* $\mathbf{Q} = \{q_i, i \in J\}$ *is the set of discrete states;*
- $\Upsilon = \bigcup_{i \in J^o} \{p_i\} \times \mathbb{R}^p$ *is the hybrid output space;* $J^o = \{1, 2, \ldots, N^o\}$, $N^o \leq N$*;* $\mathbf{P} = \{p_h, h \in J^o\}$ *is the set of discrete outputs;*
- $\Theta = \mathbf{W} \times \mathbb{R}^m$ *is the hybrid input space;* $\mathbf{W} = \mathbf{U} \cup \mathbf{V}$*, where* $\mathbf{U}$ *is the finite set of discrete control inputs,* $\mathbf{V}$ *is the finite set of discrete disturbances and* $\mathbf{U} \cap \mathbf{V} = \varnothing$*;*
- $\mathbf{S}$ *is a subclass of linear, continuous time dynamical systems. The system* $S_h \in \mathbf{S}$ *is defined by the equations:*

$$\begin{aligned} \dot{x}(t) &= A_h x(t) + B_h u(t) \\ y(t) &= C_h x(t) \end{aligned}$$

  with $h \in J$, $A_h \in \mathbb{R}^{n_h \times n_h}$, $B_h \in \mathbb{R}^{n_h \times m}$, $C_h \in \mathbb{R}^{p \times n_h}$*;*
- $S : \mathbf{Q} \to \mathbf{S}$ *is a mapping that associates a continuous time dynamical system to every discrete state; for simplicity* $S(q_i) = S_i$.
- $E \subset \mathbf{Q} \times \mathbf{W} \times \mathbf{Q}$ *is a collection of discrete transitions; we assume that if* $(q_i, \sigma, q_j) \in E$ *then* $q_i \neq q_j$.
- $R : E \times \Xi \to \Xi$ *is the linear reset function, i.e. given* $e = (q_i, \sigma, q_j) \in E$ *and* $\xi = (q_i, x) \in \Xi$, $R(e, \xi) = (q_j, M_{i,j} x)$, $M_{i,j} \in \mathbb{R}^{n_j \times n_i}$.

The triple $(\mathbf{Q}, \mathbf{W}, E)$ can be viewed as a Finite State Machine (FSM) having state set $\mathbf{Q}$ and transitions defined by E. This FSM characterizes the structure of the discrete transitions and w.l.o.g. is supposed to be connected.

The systems in $\mathcal{H}$, with $\mathbf{U} = \emptyset$ and $\mathbf{V} \neq \emptyset$ are often referred in the literature as *switching systems* [2], while systems with $\mathbf{V} = \emptyset$ and $\mathbf{U} \neq \emptyset$ are called *switched systems [15].*

Following [9], we recall that a hybrid time basis τ is an infinite or finite sequence of sets $I_j = \{t \in \mathbb{R} : t_j \leq t \leq t'_j\}$, with $t'_j = t_{j+1}$; let $card(\tau) = L + 1$. If $L < \infty$, then t'_L can be finite or infinite. Denote by $\mathcal{T}$ the set of all hybrid time bases. The hybrid system temporal evolution is then defined as follows:

Definition 2 *(Hybrid System Execution) An execution χ of a hybrid system $\mathcal{S}$ is a collection $\chi = (\xi_0, \tau, \sigma, u, \xi, \eta)$ with $\xi_0 = (\widehat{q}, x_0) \in \Xi$, $\tau \in \mathcal{T}$, $\sigma : \mathbb{N} \to \mathbf{W}$, $u \in \mathcal{U}$, $\xi : \mathbb{R} \times \mathbb{N} \to \Xi$, $\eta : \mathbb{R} \times \mathbb{N} \to \Upsilon$. Setting $\xi(t, j) = (q(j), x(t, j))$, $\forall t \in I_j$ the function ξ is defined as follows:*

$$\begin{aligned} \xi(t_0, 0) &= \xi_0 = (\widehat{q}, x_0); \\ \xi(t_{j+1}, j+1) &= R\left(e_j, \xi\left(t'_j, j\right)\right); \\ e_j &= (q(j), \sigma(j), q(j+1)) \in E; \\ x(t, j) &= x(t) \end{aligned}$$

where $x(t)$ is the (unique) solution at time t of the dynamical system $S_h = S(q(j))$, with initial condition $x(t_j) = x(t_j, j)$. Setting $\eta(t, j) = (p(j), y(t, j))$, $\forall t \in I_j$, the function η is defined as follows:

$$\begin{aligned} p(j) &= \gamma(q(j)); \\ y(t, j) &= C_h x(t, j), \; q(j) = q_h \end{aligned}$$

where $\gamma : \mathbf{Q} \to \mathbf{P}$ is a mapping that associates a discrete output to each discrete state.

We will say that $\chi = (\xi_0, \tau, \sigma, u, \xi, \eta)$ is an execution of $\mathcal{S}$ with initial state $\xi_0 \in \Xi$. The sequence σ will be called admissible with respect to the initial discrete state $\widehat{q}$.

Throughout the paper we make the following assumptions:

Assumption 1 (Minimum and maximum dwell time) Given the hybrid system $\mathcal{S}$, $0 < \delta_m \leq t'_j - t_j \leq \delta_M$, $\forall j = 0, 1, ..., L$, for any hybrid execution where δ_m is the minimum dwell time [10] and δ_M is the maximum dwell time. The value δ_M can be finite or infinite.

Assumption 1 implies that all executions of $\mathcal{H}$ are non-Zeno. Then, if there is no maximum dwell time, i.e. $\delta_M = \infty$, all executions may be assumed w.l.o.g. to be infinite. If there is a finite maximum dwell time, i.e. $\delta_M < \infty$, we assume that any execution is non-blocking as follows:

Assumption 2 Given the hybrid system $\mathcal{S}$, the FSM $\mathcal{D}_S$ is alive [11], i.e. for any $q \in \mathbf{Q}$ there exist $\sigma \in \mathbf{U}_D$ and $q' \in \mathbf{Q}$ such that $(q, \sigma, q') \in E$.

The hybrid system $\mathcal{S}$ is said to be alive if $\mathcal{D}_{\mathcal{S}}$ is alive. Under Assumptions 1 and 2, any execution of $\mathcal{S}$ may be assumed w.l.o.g. to be infinite.

Due to the switching nature of the systems we are considering and to the assumption on minimum and maximum dwell time, in general there is no time-invariant state feedback-control law that stabilizes a given system in $\mathcal{H}$. The following example better illustrates this phenomenon.

Example 3 *Let $\mathcal{S}$ be a linear hybrid system with $\mathbf{Q} = \mathbf{P} = \{q_1, q_2\}$, $\Xi = \Upsilon = \mathbf{Q} \times \mathbb{R}$, $\mathbf{U} = \{\sigma_1\}$, $\mathbf{V} = \{\sigma_2\}$, $\mathbf{S} = \{S_1, S_2\}$, with S_1 and S_2 described by the equations $\dot{x}(t) = \alpha x(t)$, $\dot{x}(t) = -\alpha x(t)$, $\alpha > 0$, respectively, $E = \{(q_1, \sigma_1, q_2), (q_2, \sigma_2, q_1)\}$ and $M_{1,2} = M_{2,1} = 1$. The only control law such that, for any initial state, the continuous component of the state remains bounded, is the function that returns the event σ_1 at time $t_j + \delta_m$, being $q(j) = q_1$, and the null event otherwise.*

In the next section, we will formally define stabilizability of a linear hybrid system by means of a control law belonging to the class of time varying state feedback functions. For the sake of simplicity, in what follows a time-varying state feedback law will be called simply state feedback law.

Let the symbol ϵ denote the null event. We can now describe the controlled execution of the hybrid system.

Definition 4 *(Hybrid Closed Loop Execution) A state feedback function for the hybrid system $\mathcal{S} \in \mathcal{H}$ is a function $\varphi : \Xi \times \mathbb{R} \to (\mathbf{U} \cup \{\epsilon\}) \times \mathbb{R}^m$. Let φ_c and φ_d denote the continuous and the discrete component of φ, respectively. A closed loop execution of $\mathcal{H}$ is a hybrid execution where given $j \in \{0, 1, ..., L\}$ and $t \in I_j$*

- $u(t) = \varphi_c(\xi(t,j), t - t_j)$;
- *if* $\varphi_d(\xi(t,j), t - t_j) \neq \epsilon$, *then* $t'_j = t$ *and*

$$\sigma(j+1) \in \{\varphi_d(\xi(t,j), t - t_j)\} \cup \mathbf{V}|_{q(j)},$$

where $\mathbf{V}|_{q(j)} = \{\delta \in \mathbf{V} : (q(j), \delta, q') \in E,$ *for some* $q' \in \mathbf{Q}\}$.

If $\varphi_d(\xi(t,j), t - t_j) \in \mathbf{U}$ the value for $\sigma(j+1)$ is set to the value of the function φ_d itself or a value that corresponds to a discrete disturbance. This is compatible with the discrete component of the current hybrid state since in the case where a discrete disturbance and a discrete control act at the same time, we assume that the switching transition (i.e. the transition due to disturbance) has priority.

3 Discrete state space decomposition

3.1 Stabilizability

Let $\mathcal{B} = \bigcup_{i \in J} \{q_i\} \times \mathcal{B}_i \subset \Xi$, $\mathcal{B}_i = \{x \in \mathbb{R}^{n_i} : \|x\| \leq 1\}$.

Definition 5 *Given $\mathbf{Q}_0 \subset \mathbf{Q}$, let $\mathcal{B}_0 = \bigcup_{i \in J_0} \{q_i\} \times \mathcal{B}_i \subset \Xi$, $J_0 = \{i \in J : q_i \in \mathbf{Q}_0\}$. A system $\mathcal{S} \in \mathcal{H}$ is $\mathbf{Q}_0-$stabilizable by state feedback if for all $\varepsilon > 0$ there exist $\rho > 0$ and a state feedback function φ such that for any $\xi_0 \in \rho\mathcal{B}_0$, $\xi(t,j) \in \varepsilon\mathcal{B}$, $\forall t \geq 0$, $\forall j \geq 0$, for all closed loop executions of the switching system with initial state ξ_0. A system $\mathcal{S} \in \mathcal{H}$ is $\mathbf{Q}_0-$ asymptotically stabilizable by state feedback if it is $\mathbf{Q}_0-$ stabilizable by state feedback and the state feedback function φ is such that, for any arbitrarily small $\varepsilon > 0$ and for any $\xi_0 \in \alpha\mathcal{B}_0$, with arbitrarily large α, a finite time $\mathbf{t}$ exists such that $\xi(t,j) \in \varepsilon\mathcal{B}$, $\forall t \geq \mathbf{t}$, $\forall j \geq \mathbf{j}$, for all closed loop executions of the hybrid system with initial state ξ_0, where $\mathbf{j} = \inf j : \mathbf{t} \in I_j$.*

If $\mathbf{Q}_0 = \mathbf{Q}$, the system is called stabilizable (asymptotically stabilizable) by state feedback.

For shortness, in the sequel we will omit the specification "by state feedback" for the stabilizability property.

The discrete transitions of the system $\mathcal{S}$ are described by a connected FSM $(\mathbf{Q}, \mathbf{W}, E)$ that can be decomposed into its strongly connected components (i.e. maximal sets of mutually reachable states), denoted as $(\mathbf{Q}_j, \mathbf{W}, E_j)$, $j = 1, 2, .., f$. The systems $\mathcal{S}_j = \left(\Xi|_{\mathbf{Q}_j}, \Upsilon, \Theta, \mathbf{S}, S|_{\mathbf{Q}_j}, E_j, R|_{\mathbf{Q}_j}\right)$, $j = 1, 2, .., f$ are called strongly connected components of $\mathcal{S}$. A leaf is a strongly connected component $\mathcal{S}_j$ of $\mathcal{S}$ with no successors, i.e. for any $q \in \mathbf{Q}_j$, $(q, \sigma, q_h) \in E$ only if $q_h \in \mathbf{Q}_j$. A proper strongly connected component is a strongly connected component with more than one discrete state.

Our first result shows that stabilizability of a given linear hybrid system depends on a suitable defined subsystem, where all the continuous dynamics are not controllable.

Given $\mathcal{S} = (\Xi, \Upsilon, \Theta, \mathbf{S}, S, E, R)$, let $\mathbf{Q}' \subset \mathbf{Q}$ denote the set of all discrete states q such that system $S(q)$ is controllable and let $\mathbf{Q}_c \subset \mathbf{Q}$ be the set such that, for some state feedback function, any closed loop execution of $\mathcal{S}$ with discrete initial state in $\mathbf{Q}_c$ has the discrete component of the hybrid state in $\mathbf{Q}'$, for some $j \in \{0, 1, \ldots, L\}$; obviously, $\mathbf{Q}' \subset \mathbf{Q}_c$. The set $\mathbf{Q}_c$ can be easily computed using the following algorithm:

Algorithm 6 *Initialize $\mathbf{Q}_c = \mathbf{Q}'$;*

- STEP 1. Let $R(\mathbf{Q}_c)$ be the set of q_i such that
  - $(q_i, \sigma, q_j) \in E$, for some $q_j \in \mathbf{Q}_c$, for some $\sigma \in \mathbf{U}$, and
  - if $(q_i, \sigma, q_j) \in E$, for some $\sigma \in \mathbf{V}$, then $q_j \in \mathbf{Q}_c$;
- STEP 2. If $R(\mathbf{Q}_c) = \mathbf{Q}_c$ then EXIT otherwise $\mathbf{Q}_c = \mathbf{Q}_c \cup R(\mathbf{Q}_c)$ and go to STEP 1.

Let $\widehat{\mathbf{Q}}$ denote the complement of $\mathbf{Q}_c$ in $\mathbf{Q}$ and let $\widehat{\mathcal{S}}$ be the system $\left(\Xi|_{\widehat{\mathbf{Q}}}, \Upsilon, \Theta, \mathbf{S}, S|_{\widehat{\mathbf{Q}}}, E|_{\widehat{\mathbf{Q}}}, R|_{\widehat{\mathbf{Q}}}\right)$. If $\delta_M < \infty$, we can define a system $\mathcal{S}" = \left(\Xi|_{\mathbf{Q}"}, \Upsilon, \Theta, \mathbf{S}, S|_{\mathbf{Q}"}, E|_{\mathbf{Q}"}, R|_{\mathbf{Q}"}\right)$, where $\mathbf{Q}"$ denotes the set of discrete states that do not belong to any proper strongly connected component.

Theorem 7 *The hybrid system $\mathcal{S}$ is (asymptotically) stabilizable if and only if the system $\widehat{\mathcal{S}}$ is (asymptotically) stabilizable. If $\delta_M < \infty$, then the system $\mathcal{S}$ is (asymptotically) stabilizable if and only if $\mathcal{S}"$ is (asymptotically) stabilizable.*

Proof: If the discrete state in some time interval is in $\mathbf{Q}_c$, then it is possible to steer the discrete state in $\mathbf{Q}'$. Since the continuous dynamics associated to $\mathbf{Q}'$ are controllable, then there exists a state feedback function for the hybrid system such that the continuous state coincides with the origin, after any arbitrarily small interval of time. Therefore, the stabilizability of $\mathcal{S}$ depends on the system $\widehat{\mathcal{S}}$. The second part of the statement is obvious. ∎

Clearly, given $\mathbf{Q}_0 \subset \mathbf{Q}$, the conditions in the theorem above are sufficient for $\mathbf{Q}_0-$ (asymptotic) stabilizability, but they are not necessary.

The last two results of this section apply to the subclasses of switching systems and switched systems, respectively.

Theorem 8 *A switching system $\mathcal{S}$ is (asymptotically) stabilizable if and only if*

- (case $\delta_M = \infty$) each strongly connected component is (asymptotically) stabilizable;

- (case $\delta_M < \infty$) each proper strongly connected component is (asymptotically) stabilizable.

Theorem 9 *A switched system $\mathcal{S}$ is stabilizable (asymptotically stabilizable, resp.) if and only if each leaf of $\mathcal{S}$ is stabilizable (asymptotically stabilizable).*

The proofs of these statements are quite simple (and hence, omitted): they are based on the fact that there is a partial ordering among the strongly connected components of the FSM and these strongly connected components determine a directed acyclic graph.

3.2 Detectability

In this section, we focus on detectability. As discussed in [5], this property is related to the existence of a suitable input function that allows the asymptotic estimation of the current hybrid state. Since in general for the class $\mathcal{H}$ the admissible values of the discrete input at some time depend on the discrete state at that time and the discrete state is supposed to be not known, we restrict our analysis to the class of switching systems.

Given a switching system $\mathcal{S} \in \mathcal{H}$ and an execution χ, consider the function $y_o : \mathbb{R} \to \Upsilon$, with $\eta_o(t) = \eta(t, j)$, $t \in [t_j, t'_j)$, $j = 0, 1, ..., L$. The restriction of η_o to the interval $[t_0, t]$ is said to be the *observed output* at time t of the switching system $\mathcal{S}$. Let $\mathcal{Y}_o$ the class of piecewise continuous functions $y_o : \mathbb{R} \to \Upsilon$.

In [5], we proposed a definition of detectability for switching systems based on the current state estimation from the knowledge of the observed output.

Definition 10 *[5] Given $\Xi_0 \subset \Xi$, a switching system $\mathcal{S} = (\Xi, \Upsilon, \Theta, \mathbf{S}, S, E, R) \in \mathcal{H}$ is Ξ_0-detectable if there exist a function $\vartheta : \mathcal{Y}_o \times \mathcal{U} \to \Xi$ and a real $\Delta \in (0, \delta_m)$ such that, by setting $\vartheta(.) = (\vartheta_{\mathbf{Q}}(.), \vartheta_{\mathbb{R}}(.))$, where $\vartheta_{\mathbf{Q}}(.) = q_i \in \mathbf{Q}$ and $\vartheta_{\mathbb{R}}(.) \in \mathbb{R}^{n_i}$, $\forall \xi_0 = (\widehat{q}, x_0) \in \Xi_0, \forall \tau \in \mathcal{T}$, $\forall \sigma$ admissible w.r.t. $\widehat{q}$, $\forall \varepsilon > 0$, there exist an execution $\chi = (\xi_0, \tau, \sigma, u, \xi, \eta)$ and $\mathbf{t} > t_0$ such that*

$$
\begin{array}{ll}
(i) & \vartheta_{\mathbf{Q}}\left(y_o|_{[t_0,t]}, u|_{[t_0,t)}\right) = q(j); \\
(ii) & \left\| \vartheta_{\mathbb{R}}\left(y_o|_{[t_0,t]}, u|_{[t_0,t)}\right) - x(t,j) \right\| \leq \varepsilon.
\end{array}
$$

$\forall t \in [\mathbf{t}, \infty) \cap [t_j + \Delta, t'_j], \forall j = \mathbf{j}, ..., L,$ *where* $\mathbf{j} = \inf j : \mathbf{t} \in I_j$.

If $\Xi_0 = \Xi$, the system is called detectable.

Informally, the above definition means that there exists at least an input-output experiment such that

- after some switchings, the discrete component of the state is exactly reconstructed;
- after some time that depends on the required precision, the continuous component of the state is estimated, with a fixed maximum error.

If the function ϑ and the real $\Delta \in (0, \delta_m)$ are such that $\vartheta_{\mathbf{Q}}\left(y_o|_{[t_0,t]}, u|_{[t_0,t)}\right) = q(j)$, for any $j = 0, 1, ..., L$

and for any $t \in [t_j+\Delta, t'_j]$, then we say that the discrete component of the hybrid state can be reconstructed for any I_j. Thanks to results developed in [5], the following holds.

Lemma 11 *Given a switching system $\mathcal{S}$, the discrete state is reconstructable for any I_j if and only if $\forall p \in \mathcal{R}(\gamma)$, $\forall q_i, q_j \in \gamma^{-1}(p)$, $\exists k \in \mathbb{N} \cup \{0\} : C_i A_i^k B_i \neq C_j A_j^k B_j$.*

If a switching system is detectable, as in Definition 10, it can be proved that it is detectable for any input function in a subclass of $\mathcal{U}$, having the property that its complement in $\mathcal{U}$ is "thin" [5]. Therefore Definition 10 can be viewed as a generalization of the concept of *generic final-state determinability*, introduced in [12].

As done for stabilizability, we define an auxiliary system, whose detectability is equivalent to the detectability of the given system. Given the discrete state space $\mathbf{Q}$, let $\mathbf{Q}^o \subset \mathbf{Q}$ denote the set of all discrete states q such that system $S(q)$ is observable and let $\overline{\mathbf{Q}}$ denote the complement of $\mathbf{Q}^o$ in $\mathbf{Q}$. We can define a switching system $\overline{\mathcal{S}} = \left(\Xi|_{\overline{\mathbf{Q}}}, \Upsilon, \Theta, \mathbf{S}, S|_{\overline{\mathbf{Q}}}, E|_{\overline{\mathbf{Q}}}, R|_{\overline{\mathbf{Q}}}\right)$; the symbol $\mathcal{S}$" has already been defined in the previous subsection. The proofs of the following theorems are omitted; they are based on arguments similar to those used in the proofs of Theorem 8 and Theorem 9.

Theorem 12 *Let us assume that the discrete component of the hybrid state can be reconstructed for any I_j. The hybrid system $\mathcal{S}$ is detectable if and only if the system $\overline{\mathcal{S}}$ is detectable. If $\delta_M < \infty$, then the system $\mathcal{S}$ is detectable if and only if $\mathcal{S}$" is detectable.*

Finally,

Theorem 13 *Let us assume that the discrete component of the hybrid state can be reconstructed for any I_j. A hybrid system $\mathcal{S}$ is detectable if and only if*

- (case $\delta_M = \infty$) each strongly connected component is detectable;

- (case $\delta_M < \infty$) each proper strongly connected component is detectable.

It was proved in [5] that in the case of switching systems with $\delta_M = \infty$, a necessary condition for detectability is that the discrete component of the hybrid state can be reconstructed for any I_j from the knowledge of the observed output. Therefore, for the class of switching systems, the assumptions of the theorems above can be made without loss of generality.

4 Conclusions

We presented results that improve the efficiency in verifying stabilizability and detectability of linear hybrid systems. Our approach is based on a decomposition of the discrete state space, which allows to reduce the complexity of the verification problem that is mostly due to the presence of cycles in the structure of the finite state machine part of the hybrid system.

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