Preprint Proceedings 6th Int. Conf on Information Fusion, 8-11 July 2003, Cairns, Queensland, Australia

Joint IMMPDA Particle filter

Henk A.P. Blom and Edwin A. Bloem
National Aerospace Laboratory NLR
Amsterdam, The Netherlands
e-mail: blom@nlr.nl, bloem@nlr.nl

Abstract – *For the problem of tracking multiple manoeuvering targets in clutter and missing measurements the paper develops a Joint IMMPDA type of particle filter and compares this with other IMMJPDA based filters through Monte Carlo simulation for a simple example.*

Keywords: Bayesian estimation, Multitarget tracking, Sudden maneuvers, Clutter, Missed detections, Hidden Markov model, Descriptor system, Particle filtering.

1 Introduction

McGinnity & Irwin (2000, 2001), Doucet et al. (2001) and Musso et al. (2001) showed that estimation of jump linear systems with particle filter approaches has certain performance advantages over the Interacting Multiple Model algorithm (Blom, 1984; Blom & Bar-Shalom, 1988). Similarly, for the problem of tracking multiple targets in clutter and missed detections, Avitzour (1995) and Gordon (1997) have reported that particle filters outperform Gaussian density approximations of Bayesian filters using the bootstrap approach of Gordon et al. (1993). The aim of this paper is to extend the bootstrap particle filtering approach of McGinnity & Irwin (2000) to situations of possibly false and missing observations of multiple maneuvering targets.
Following Blom & Bloem (2002a, 2002b) this multitarget tracking problem is first presented as one of filtering for a descriptor system with both i.i.d. and Markovian coefficients. For this descriptor system we develop a characterization of the evolution of the exact conditional density function. The specialty of this exact equation is that both the IMM step and the PDA step are performed jointly for all targets. In contrast with this the IMMJPDA of Chen & Tugnait (2001) jointly performs the PDA step only. Following the exact equations, we develop a Joint IMMPDA Particle (JIMMPDAP) which evaluates the exact equations through the particle filtering approach of McGinnity & Irwin (2000). Through Monte Carlo simulations for a simple example this novel algorithm is compared with the IMMJPDA of Chen & Tugnait (2001) and the track coalescence avoiding IMMJPDA* of Blom & Bloem (2002a,b).

The paper is organized as follows. Section 2 formulates the problem considered. In this way it is ensured that there is no unambiguity which mathematical model is addressed. Section 3 develops an exact Bayesian characterization of the evolution of the conditional density for the state of the multiple targets. Section 4 develops the JIMMPDA Particle filter. Section 5 shows the effectiveness of this filter through Monte Carlo simulation results. Finally, Section 6 draws conclusions.

[0]This research has been performed with support of the European Commission through the HYBRIDGE project.

2 Problem formulation

Following Blom & Bloem (2002a,b) the problem is formulated in terms of filtering for a jump linear descriptor system with both Markovian switching and i.i.d. coefficients:

$$x_{t+1} = A(\theta_{t+1})x_t + B(\theta_{t+1})w_t \tag{1}$$

$$\underline{\Phi}(\psi^*_t)\mathrm{y}_t = \quad v^*_t \quad \text{if } L_t > D_t, \tag{2}$$

$$\underline{\Phi}(\psi_t)\mathrm{y}_t = \underline{\chi}_t\underline{\Phi}(\phi_t)[H(\theta_t)x_t + G(\theta_t)v_t] \text{ if } D_t > 0 \tag{3}$$

Target evolution eq. (1)
The underlying model components are follows:
$x_t \triangleq \text{Col}\{x^1_t, ..., x^M_t\}$,
$\theta_t \triangleq \text{Col}\{\theta^1_t, ..., \theta^M_t\}$,
$A(\theta) \triangleq \text{Diag}\{a^1(\theta^1), ..., a^M(\theta^M)\}$,
$B(\theta) \triangleq \text{Diag}\{b^1(\theta^1), ..., b^M(\theta^M)\}$,
$w_t \triangleq \text{Col}\{w^1_t, ..., w^M_t\}$,
where x^i_t is the n-vectorial state of the i-th target at

moment t, θ_t^i is the mode of the i-th target at moment t and assumes values from $\mathbb{M} = \{1, .., N\}$, $a^i(\theta^i)$ and $b^i(\theta^i)$ are $(n \times n)$- and $(n \times n')$-matrices, w_t^i is a sequence of i.i.d. standard Gaussian variables of dimension n' with w_t^i , w_t^j independent for all $i \neq j$ and w_t^i ,x_0^i, x_0^j independent for all $i \neq j$.

Clutter measurements eq. (2)

$\mathrm{y}_t \stackrel{\triangle}{=} \mathrm{Col}\{\mathrm{y}_{1,t}, ..., \mathrm{y}_{L_t,t}\}$ is the measurement vector that contains a random mixture of target- and clutter measurements, where $\mathrm{y}_{i,t}$ denotes the i-th m-vectorial measurement at moment t, and L_t is the number of measurements at moment t.

v_t^* is a column vector of $L_t - D_t$ i.i.d. false measurements with density $p_{v_t^*|F_t}(v^*|F) = V^{-F}$, where F refers to a number of false measurements in volume V.

$\psi_t^* \stackrel{\triangle}{=} \mathrm{Col}\{\psi_{1,t}^*, ..., \psi_{L_t,t}^*\}$ is a clutter indicator vector of size L_t with $\psi_{l,t}^* \in [0, 1]$ the clutter indicator at moment t for measurement i. It assumes the value one if measurement i comes from clutter and zero if measurement i belongs to a target.

In order to select the clutter measurements by simple matrix multiplication, a matrix operator Φ is defined, producing $\Phi(\psi')$ as a $(0, 1)$-valued matrix of size $D(\psi') \times M'$ of which the ith row equals the ith non-zero row of $\mathrm{Diag}\{\psi'\}$, where $D(\psi') \stackrel{\triangle}{=} \sum_{i=1}^{M'} \psi_i'$ for an arbitrary (0,1)-valued M'-vector ψ'. To take into account the measurement vector size m, $\Phi(\psi_t^*)$ needs to be "inflated" to the proper size of $D_t m$ by means of the tensor product with I_m. To this end, $\underline{\Phi}(\psi') \stackrel{\triangle}{=} \Phi(\psi') \otimes I_m$ with I_m a unit-matrix of size m, and $\otimes$ the tensor product. Hence $\underline{\Phi}(\psi_t^*)\mathrm{y}_t$ is a column vector that contains only clutter measurements from y_t.

Notice that the order of the clutter measurements in (3) is not of interest.

Target measurement eq. (3)

The coefficients on the right hand side of eq. (2) are:

$H(\theta) \stackrel{\triangle}{=} \mathrm{Diag}\{h^1(\theta^1), ..., h^M(\theta^M)\}$,

$G(\theta) \stackrel{\triangle}{=} \mathrm{Diag}\{g^1(\theta^1), ..., g^M(\theta^M)\}$,

$h^i(\theta^i)$ is an $(m \times n)$-matrix,

$g^i(\theta^i)$ is an $(m \times m')$-matrix,

$v_t \stackrel{\triangle}{=} \mathrm{Col}\{v_t^1, ..., v_t^M\}$,

where v_t^i is a sequence of i.i.d. standard Gaussian variables of dimension m' with v_t^i and v_t^j independent for all $i \neq j$. Moreover v_t^i is independent of x_0^j and w_t^j for all i,j.

$\psi_t \stackrel{\triangle}{=} \mathrm{Col}\{\psi_{1,t}, ..., \psi_{L_t,t}\}$ is the target indicator vector, where $\psi_{i,t} \in$ {0,1} is a target indicator at moment t for measurement i, which assumes the value one if measurement i belongs to a detected target and zero if measurement i comes from clutter.

To select the target measurements, which are indicated by the target indicator vector, by simple matrix multiplication, the matrix operator Φ is used again. Hence $\underline{\Phi}(\psi_t)\mathrm{y}_t$ is a column vector that contains target measurements from y_t only, in a random order.

$\phi_t \stackrel{\triangle}{=} \mathrm{Col}\{\phi_{1,t}, ..., \phi_{M,t}\}$ is the detection indicator vector, where $\phi_{i,t} \in$ {0,1} is the detection indicator for target i, which assumes the value one with probability $P_{\mathrm{d}}^i > 0$, independently of $\phi_{j,t}$, $j \neq i$, where P_{d}^i denotes the detection probability of target i. $\{\phi_t\}$ is a sequence of i.i.d. vectors, and $D_t \stackrel{\triangle}{=} \sum_{i=1}^{M} \phi_{i,t}$ denotes the number of detected targets. Hence $L_t - D_t$ is the number of clutter measurements. As before, by using the matrix operator Φ, $\underline{\Phi}(\phi_t)H(\theta_t)x_t$ is a column vector of potential detected measurements of targets in a fixed order.

Finally the detected target measurements in the observation vector y_t are in random order. Hence the potential detected measurements of targets need to be randomly mixed. To perform this by a simple matrix multiplication, a sequence of independent stochastic permutation matrices $\{\chi_t\}$ of size $D_t \times D_t$ is defined and assumed to be independent of $\{\phi_t\}$. To take into account the measurement vector size m, χ_t needs to be "inflated" to the proper size of $D_t m$ by means of the tensor product with I_m. To this end, $\underline{\chi}_t \stackrel{\triangle}{=} \chi_t \otimes I_m$ with I_m a unit-matrix of size m, and $\otimes$ the tensor product. Hence $\underline{\chi}_t\underline{\Phi}(\phi_t)H(\theta_t)x_t$ is a column vector of potential detected measurements of targets in random order.

3 Exact filter equations

In this section a Bayesian characterization of the conditional density $p_{x_t,\theta_t|\ Y_t}(x, \theta)$ is given where Y_t denotes the σ-algebra generated by measurements y_t up to and including moment t. Subsequently, characterizations are developed for the mode probabilities and the mode conditional means and covariances.

Notice that (2) is a linear Gaussian descriptor system (Dai, 1989) with stochastic i.i.d. coefficients $\underline{\Phi}(\psi_t)$ and $\underline{\chi}_t\underline{\Phi}(\phi_t)$ and Markovian switching coefficients $H(\theta_t)$ and $G(\theta_t)$. Because χ_t has an inverse, (2) can be transformed into

$$\underline{\chi}_t^T \underline{\Phi}(\psi_t)\mathrm{y}_t = \underline{\Phi}(\phi_t)H(\theta_t)x_t + \underline{\Phi}(\phi_t)G(\theta_t)v_t, \text{ if } D_t > 0 \quad (4)$$

Next we introduce an auxiliary indicator matrix process $\tilde{\chi}_t$ of size $D_t \times L_t$, as follows:

$$\tilde{\chi}_t \stackrel{\triangle}{=} \chi_t^T \Phi(\psi_t) \text{ if } D_t > 0. \quad (5.a)$$

and an auxiliary measurement process

$$\tilde{\mathrm{y}}_t \stackrel{\triangle}{=} \tilde{\underline{\chi}}_t \mathrm{y}_t \quad (5.b)$$

With this we get a simplified version of (4):

$$\mathrm{y}_t = \tilde{\underline{\chi}}_t \mathrm{y}_t = \underline{\Phi}(\phi_t)H(\theta_t)x_t + \underline{\Phi}(\phi_t)G(\theta_t)v_t, \text{ if } D_t > 0, \quad (6)$$

where the size of $\underline{\tilde{\chi}}_t$ is $D_t m \times L_t m$ and the size of $\underline{\Phi}(\phi_t)$ is $D_t m \times Mm$.
From (6), it follows that for $D_t > 0$ all relevant associations and permutations can be covered by $(\phi_t, \tilde{\chi}_t)$-hypotheses. We extend this to $D_t = 0$ by adding the combination $\phi_t = \{0\}^M$ and $\tilde{\chi}_t = \{\}^{L_t}$. Hence, through defining the weights

$$\beta_t(\phi, \tilde{\chi}, \theta) \triangleq \text{Prob}\{\phi_t = \phi, \tilde{\chi}_t = \tilde{\chi}, \theta_t = \theta \mid Y_t\},$$

the law of total probability yields:

$$p_{x_t\theta_t|Y_t}(x, \theta) = \sum_{\tilde{\chi},\phi} \beta_t(\phi, \tilde{\chi}, \theta) p_{x_t|\theta_t,\phi_t,\tilde{\chi}_t,Y_t}(x \mid \theta, \phi, \tilde{\chi}) \tag{7}$$

And thus, our problem is to characterize the terms in the last summation. This problem is solved in two steps, the first of which is the following Theorem.

Theorem 1. *For any $\phi \in \{0,1\}^M$, such that $D(\phi) \triangleq \sum_{i=1}^M \phi_i \leq L_t$, and any $\tilde{\chi}_t$ matrix realization $\tilde{\chi}$ of size $D(\phi) \times L_t$, the following holds true:*

$$p_{x_t|\theta_t,\phi_t,\tilde{\chi}_t,Y_t}(x \mid \theta, \phi, \tilde{\chi}) = \\ = \frac{p_{\tilde{y}_t|x_t,\theta_t,\phi_t}(\underline{\tilde{\chi}} y_t \mid x, \theta, \phi) \cdot p_{x_t|\theta_t,Y_{t-1}}(x \mid \theta)}{F_t(\phi, \tilde{\chi}, \theta)} \tag{8}$$

$$\beta_t(\phi, \tilde{\chi}, \theta) = F_t(\phi, \tilde{\chi}, \theta) \lambda^{(L_t - D(\phi))} \cdot \\ \cdot [\prod_{i=1}^M (1 - P_d^i)^{(1-\phi_i)} (P_d^i)^{\phi_i}] \cdot p_{\theta_t|Y_{t-1}}(\theta)/c_t \tag{9}$$

where $\underline{\tilde{\chi}} \triangleq \tilde{\chi} \otimes I_m$, and $F_t(\phi, \tilde{\chi}, \theta)$ and c_t are such that they normalize $p_{x_t|\theta_t,\phi_t,\tilde{\chi}_t,Y_t}(x \mid \theta, \phi, \tilde{\chi})$ and $\beta_t(\phi, \tilde{\chi}, \theta)$ respectively.

Proof: If $\phi = 0$ we get $p_{x_t|\theta_t,\phi_t,\tilde{\chi}_t,Y_t}(x \mid \theta, 0, \tilde{\chi}) = p_{x_t|\theta_t,Y_{t-1}}(x \mid \theta)$. Else

$$p_{x_t|\theta_t,\phi_t,\tilde{\chi}_t,Y_t}(x \mid \theta, \phi, \tilde{\chi}) = \\ = p_{x_t|\theta_t,\phi_t,\tilde{\chi}_t,y_t,L_t,Y_{t-1}}(x \mid \theta, \phi, \tilde{\chi}, y_t, L_t) = \\ = p_{x_t|\theta_t,\phi_t,\tilde{\chi}_t,y_t,L_t,\tilde{y}_t,Y_{t-1}}(x \mid \theta, \phi, \tilde{\chi}, y_t, L_t, \underline{\tilde{\chi}} y_t) = \\ = p_{x_t|\theta_t,\phi_t,\tilde{y}_t,Y_{t-1}}(x \mid \theta, \phi, \underline{\tilde{\chi}} y_t) = \\ = p_{\tilde{y}_t|x_t,\theta_t,\phi_t}(\underline{\tilde{\chi}} y_t \mid x, \theta, \phi) \cdot \\ \cdot p_{x_t|\theta_t,Y_{t-1}}(x \mid \theta)/F_t(\phi, \tilde{\chi}, \theta)$$

with $F_t(\phi, \tilde{\chi}, \theta) \triangleq p_{\tilde{y}_t|\theta_t,\phi_t,Y_{t-1}}(\underline{\tilde{\chi}} y_t \mid \theta, \phi)$. Subsequently

$$\beta_t(\phi, \tilde{\chi}, \theta) \triangleq \text{Prob}\{\phi_t = \phi, \tilde{\chi}_t = \tilde{\chi}, \theta_t = \theta \mid Y_t\} = \\ = p_{\phi_t,\tilde{\chi}_t,\theta_t|Y_t}(\phi, \tilde{\chi}, \theta) = \\ = p_{\phi_t,\tilde{\chi}_t\theta_t|y_t,L_t,Y_{t-1}}(\phi, \tilde{\chi}, \theta \mid y_t, L_t) = \\ = p_{y_t,\tilde{\chi}_t,\theta_t|\phi_t,L_t,Y_{t-1}}(y_t, \tilde{\chi}, \theta \mid \phi, L_t) \cdot \\ \cdot p_{\phi_t|L_t,Y_{t-1}}(\phi \mid L_t)/c'_t = \\ = p_{y_t,\tilde{\chi}_t|\theta_t,\phi_t,L_t,Y_{t-1}}(y_t, \tilde{\chi} \mid \theta, \phi, L_t) \cdot \\ \cdot p_{\phi_t|L_t,Y_{t-1}}(\phi \mid L_t) p_{\theta_t|Y_{t-1}}(\theta)/c'_t$$

If $D_t > 0$ we have

$$\begin{aligned} \tilde{\chi}_t^T \tilde{\chi}_t &= \Phi(\psi_t)^T \chi_t \chi_t^T \Phi(\psi_t) = \\ &= \Phi(\psi_t)^T \Phi(\psi_t) = \\ &= \text{Diag}\{\psi_t\} \end{aligned}$$

$$\begin{aligned} \tilde{\chi}_t \Phi(\psi_t)^T &= \chi_t^T \Phi(\psi_t) \Phi(\psi_t)^T = \\ &= \chi_t^T \end{aligned}$$

which means that the transformation from (ψ_t, χ_t) into $\tilde{\chi}_t$ has an inverse, which implies

$$p_{y_t,\tilde{\chi}_t|\theta_t,\phi_t,L_t,Y_{t-1}}(y_t, \chi^T \Phi(\psi) \mid \theta, \phi, L_t) = \\ = p_{y_t,\psi_t,\chi_t|\theta_t,\phi_t,L_t,Y_{t-1}}(y_t, \psi, \chi \mid \theta, \phi, L_t)$$

Furthermore, because the transformation from (y_t, ψ_t, χ_t) into $(\tilde{y}_t, v_t^*, \psi_t, \chi_t)$ is a permutation, we get for $L_t > D(\phi) > 0$

$$p_{y_t,\psi_t,\chi_t|\theta_t,\phi_t,L_t,Y_{t-1}}(y_t, \psi, \chi \mid \theta, \phi, L_t) = \\ = p_{\tilde{y}_t,v_t^*,\psi_t,\chi_t|\theta_t,\phi_t,L_t,Y_{t-1}}(\underline{\chi}^T \underline{\Phi}(\psi) y_t, \underline{\Phi}(\psi^*) y_t, \psi, \chi | \theta, \phi, L_t)$$

Hence, for $L_t > D(\phi) > 0$, β_t satisfies:

$$\beta_t(\phi, \chi^T \Phi(\psi), \theta) = F_t(\phi, \chi^T \Phi(\psi), \theta) \cdot \\ \cdot p_{v_t^*|\phi_t,L_t}(\underline{\Phi}(\psi^*) y_t \mid \phi, L_t) p_{\psi_t|\phi_t,L_t}(\psi \mid \phi, L_t) \cdot \\ \cdot p_{\chi_t|\phi_t}(\chi \mid \phi) p_{L_t|\phi_t}(L_t \mid \phi) p_{\phi_t}(\phi) p_{\theta_t|Y_{t-1}}(\theta)/c''$$

Subsequently using the JPDA derivation [2] yields:

$$\beta_t(\phi, \chi^T \Phi(\psi), \theta) = F_t(\phi, \chi^T \Phi(\psi), \theta) \lambda^{(L_t - D(\phi))} \cdot \\ \cdot [\prod_{i=1}^M (P_d^i)^{\phi_i} (1 - P_d^i)^{(1-\phi_i)}] p_{\theta_t|Y_{t-1}}(\theta)/c_t$$

with c_t a normalizing constant. It can be easily verified that the last equation also holds true if $L_t = D(\phi)$ or $D(\phi) = 0$. ■

In order to prepare for a particle filter approach, substituting (8) and (9) into (7) yields

$$p_{x_t,\theta_t|Y_t}(x, \theta) = \\ \sum_{\tilde{\chi},\phi} \frac{p_{\tilde{y}_t|x_t,\theta_t,\phi_t}(\underline{\tilde{\chi}} y_t \mid x, \theta, \phi) \cdot p_{x_t|\theta_t,Y_{t-1}}(x \mid \theta)}{F_t(\phi, \tilde{\chi}, \theta)} \cdot \\ F_t(\phi, \tilde{\chi}, \theta) \lambda^{(L_t - D(\phi))} \cdot [\prod_{i=1}^M (1 - P_d^i)^{(1-\phi_i)} (P_d^i)^{\phi_i}] \cdot \\ \cdot p_{\theta_t|Y_{t-1}}(\theta)/c_t \tag{10}$$

Simplifying (10) and rearranging terms yields:

$$p_{x_t,\theta_t|Y_t}(x, \theta) = \\ \sum_{\tilde{\chi},\phi} p_{\tilde{y}_t|x_t,\theta_t,\phi_t}(\underline{\tilde{\chi}} y_t \mid x, \theta, \phi) \cdot p_{x_t,\theta_t|Y_{t-1}}(x, \theta) \\ \lambda^{(L_t - D(\phi))} \cdot [\prod_{i=1}^M (1 - P_d^i)^{(1-\phi_i)} (P_d^i)^{\phi_i}]/c_t \tag{11}$$

with

$$p_{\tilde{y}_t|x_t,\theta_t,\phi_t}(\tilde{y} \mid x, \theta, \phi) = N\{\tilde{y}; \underline{\Phi}(\phi)H(\theta)x, \underline{\Phi}(\phi)G(\theta)G(\theta)^T\underline{\Phi}(\phi)^T\} \quad (12)$$

Define

$$\tilde{F}_t(\phi, \tilde{\chi}, x, \theta) \triangleq p_{\tilde{y}_t|x_t,\theta_t,\phi_t}(\underline{\tilde{\chi}}y_t \mid x, \theta, \phi) \quad (13)$$

Hence from (12) we get:

$$\tilde{F}_t(\phi, \tilde{\chi}, x, \theta) = [(2\pi)^{mD(\phi)}\text{Det}\{\tilde{Q}_t(\phi, \theta)\}]^{-\frac{1}{2}} \cdot \cdot \exp\{-\frac{1}{2}\tilde{\mu}_t^T(\phi, \tilde{\chi}, x, \theta)\tilde{Q}_t(\phi, \theta)^{-1}\tilde{\mu}_t(\phi, \tilde{\chi}, x, \theta)\} \quad (14)$$

where

$$\tilde{\mu}_t(\phi, \tilde{\chi}, x, \theta) \triangleq \underline{\tilde{\chi}}y_t - \underline{\Phi}(\phi)H(\theta)x$$
$$\tilde{Q}_t(\phi, \theta) \triangleq \underline{\Phi}(\phi)\big(G(\theta)G(\theta)^T\big)\underline{\Phi}(\phi)^T$$

Substituting (14) into (11) and rearranging terms yields

$$p_{x_t,\theta_t|Y_t}(x, \theta) = \frac{1}{c_t}\sum_{\tilde{\chi},\phi}\tilde{F}_t(\phi, \tilde{\chi}, x, \theta) \cdot \lambda^{(L_t - D(\phi))} \cdot \cdot [\prod_{i=1}^{M}(1 - P_d^i)^{(1-\phi_i)}(P_d^i)^{\phi_i}] \cdot p_{x_t,\theta_t|Y_{t-1}}(x, \theta) \quad (15)$$

4 Joint IMMPDA particle filter

In this section a JIMMPDA Particle filter of the exact filter characterization of Theorem 1 is developed following the approach of McGinnity & Irwin (2000). One cycle of this JIMMPDA Particle filter consists of the following seven steps, where a particle is defined as a triplet (w, x, θ), $w \in [0, 1]$, $x \in \mathbb{R}^{Mn}$, $\theta \in \mathbb{M}^M$.

JIMMPDA Particle filter Step 1: Start with the mode probabilities

$$\hat{\gamma}_{t-1}(\theta) \triangleq p_{\theta_{t-1}|Y_{t-1}}(\theta)$$

and for each $\theta \in \mathbb{M}^M$ a set of S^θ particles in $[0, 1] \times \mathbb{R}^{Mn} \times \mathbb{M}^M$, i.e.:

$$\{(w_{t-1}^{\theta,j}, x_{t-1}^{\theta,j}, \theta_{t-1}^{\theta,j} = \theta);\ j \in [1, S^\theta], \theta \in \mathbb{M}^M\}$$

with

$$w_{t-1}^{\theta,j} = \hat{\gamma}_{t-1}(\theta)/S^\theta$$

Thus in total there are $S = \sum_\theta S^\theta$ particles.

JIMMPDA Particle filter Step 2: (Interaction) Determine the new set of particles (the weights $w_{t-1}^{\theta,j}$ are not changed)

$$\{(w_{t-1}^{\theta,j}, x_{t-1}^{\theta,j}, \bar{\theta}_t^{\theta,j});\ j \in [1, S^\theta], \theta \in \mathbb{M}^M\}$$

by generating for each particle a new value $\bar{\theta}_t^{\theta,j}$ according to the model

$$\text{Prob}\{\bar{\theta}_t^{\theta,j} = \bar{\theta} \mid \theta_{t-1}^{\theta,j} = \theta\} = \Pi_{\theta,\bar{\theta}}$$

JIMMPDA Particle filter Step 3: Determine the new set of particles (the weights $w_{t-1}^{\theta,j}$ are not changed)

$$\{(w_{t-1}^{\theta,j}, \bar{x}_t^{\theta,j}, \bar{\theta}_t^{\theta,j});\ j \in [1, S^\theta], \theta \in \mathbb{M}^M\}$$

by running for each particle a Monte Carlo simulation from $(t-1)$ to t according to the model

$$\bar{x}_t^{\theta,j} = A(\bar{\theta}_t^{\theta,j})x_{t-1}^{\theta,j} + B(\bar{\theta}_t^{\theta,j})w_{t-1}$$

JIMMPDA Particle filter Step 4: Determine new weights for the set of particles, i.e.

$$\{(\bar{w}_t^{\theta,j}, \bar{x}_t^{\theta,j}, \bar{\theta}_t^{\theta,j});\ j \in [1, S^\theta], \theta \in \mathbb{M}^M\}$$

with for the new weights

$$\bar{w}_t^{\theta,j} = w_{t-1}^{\theta,j} \cdot \frac{1}{c_t}\sum_{\tilde{\chi},\phi}\tilde{F}_t(\phi, \tilde{\chi}, \bar{x}_t^{\theta,j}, \bar{\theta}_t^{\theta,j}) \cdot \lambda^{(L_t - D(\phi))} \cdot \cdot [\prod_{i=1}^{M}(1 - P_d^i)^{(1-\phi_i)}(P_d^i)^{\phi_i}]$$

where

$$\tilde{F}_t(\phi, \tilde{\chi}, x, \theta) = [(2\pi)^{mD(\phi)}\text{Det}\{\tilde{Q}_t(\phi, \theta)\}]^{-\frac{1}{2}} \cdot \cdot \exp\{-\frac{1}{2}\tilde{\mu}_t^T(\phi, \tilde{\chi}, x, \theta)\tilde{Q}_t(\phi, \theta)^{-1}\tilde{\mu}_t(\phi, \tilde{\chi}, x, \theta)\} \quad (16)$$

with

$$\tilde{\mu}_t(\phi, \tilde{\chi}, x, \theta) \triangleq \underline{\tilde{\chi}}y_t - \underline{\Phi}(\phi)H(\theta)x$$
$$\tilde{Q}_t(\phi, \theta) \triangleq \underline{\Phi}(\phi)\big(G(\theta)G(\theta)^T\big)\underline{\Phi}(\phi)^T$$

and c_t such that

$$\sum_{\theta \in \mathbb{M}^M}\sum_{j=1}^{S^\theta}\bar{w}_t^{\theta,j} = 1$$

JIMMPDA Particle filter Step 5: MMSE measurement update equations:

$$\hat{\gamma}_t(\theta) = \sum_{\eta \in \mathbb{M}^M}\sum_{j=1}^{S^\eta}\bar{w}_t^{\eta,j}1_{\bar{\theta}_t^{\eta,j}}(\theta)$$

$$\hat{x}_t(\theta) = \sum_{\eta \in \mathbb{M}^M}\sum_{j=1}^{S^\eta}\bar{w}_t^{\eta,j}\bar{x}_t^{\eta,j}1_{\bar{\theta}_t^{\eta,j}}(\theta)$$

$$\hat{P}_t(\theta) = \sum_{\eta \in \mathbb{M}^M}\sum_{j=1}^{S^\eta}\bar{w}_t^{\eta,j}[\bar{x}_t^{\eta,j} - \hat{x}_t(\theta)][\bar{x}_t^{\eta,j} - \hat{x}_t(\theta)]^T 1_{\bar{\theta}_t^{\eta,j}}(\theta)$$

JIMMPDA Particle filter Step 6: θ dependent resampling: Generate the new set of particles

$$\{(w_t^{\theta,j}, x_t^{\theta,j}, \theta_t^{\theta,j});\ j \in [1, S^\theta], \theta \in \mathbb{M}^M\}$$

by applying the following equations per θ value:

$$\theta_t^{\theta,j} = \theta$$

$$w_t^{\theta,j} = \hat{\gamma}_t(\theta)/S^\theta$$

$x_t^{\theta,j}$ is the j-th of the S^θ samples drawn from the particle spanned joint conditional density for (x_t, θ_t) given Y_t:

$$\sum_{\eta \in \mathbb{M}^M} \sum_{l=1}^{S^\eta} \bar{w}_t^{\eta,l} 1_{\bar{\theta}_t^{\eta,l}}(\theta) \delta_{\bar{x}_t^{\eta,l}}(x)$$

JIMMPDA Particle filter Step 7: MMSE output equations:

$$\hat{x}_t = \sum_{\theta \in \mathbb{M}^M} \hat{\gamma}(\theta) \hat{x}_t(\theta)$$

$$\hat{P}_t = \sum_{\theta \in \mathbb{M}^M} \hat{\gamma}(\theta)[\hat{x}_t(\theta) - \hat{x}_t][\hat{x}_t(\theta) - \hat{x}_t]^T$$

5 Monte Carlo simulations

In this section some Monte Carlo simulation results are given for the JIMMPDA Particle filter, the IMMJPDA* and IMMJPDA filter algorithms, and for an IMMPDA which updates an individual track using PDA by assuming the measurements from the adjacent targets as false. The JIMMPDA Particle filter ran on a total of $S = 10000$ particles, with for each of the four modes $S^\theta = 2500$ particles. The simulations primarily aim at gaining insight into the behavior and performance of the filters when objects move in and out close approach situations, while giving the filters enough time to converge after a manoeuvre has taken place. In the example scenarios there are two targets, each modelled with two possible modes. The first mode represents a constant velocity model and the second mode represents a constant acceleration model. Both objects start moving towards each other, each with constant initial velocity V_{initial} (i.e. the initial relative velocity $V_{\text{rel, initial}} = 2V$). At a certain moment in time both objects start decelerating with -0.5 m/s^2 until they both have zero velocity. The moment at which the deceleration starts is such that when the objects both have zero velocity, the distance between the two objects equals d (see figure 1). After spending a significant number of scans with zero velocity, both objects start accelerating with 0.5 m/s^2 away from each other without crossing until their velocity equals the opposite of their initial velocity. From that moment on the velocity of both objects remains constant again (thus the final relative velocity $V_{\text{rel, final}} = V_{\text{rel, initial}}$). Note that $d < 0$ implies that the objects have crossed each other before they have reached zero velocity. Each simulation the filters start with perfect estimates and run for 40 scans. Examples of the trajectories for $d > 0$ and $d < 0$ are depicted in figures 1a and 1b respectively.

For each target, the underlying model of the potential target measurements is given by (29) and (30)

$$x_{t+1}^i = a^i(\theta_{t+1}^i)x_t^i + b^i(\theta_{t+1}^i)w_t^i \quad (17)$$

$$z_t^i = h^i(\theta_t^i)x_t^i + g^i(\theta_t^i)v_t^i \quad (18)$$

Furthermore for $i = 1, 2$ and $\theta_t^i \in \{1,2\}$:

$$a^i(1) = \begin{bmatrix} 1 & T_s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad a^i(2) = \begin{bmatrix} 1 & T_s & \frac{1}{2}T_s^2 \\ 0 & 1 & T_s \\ 0 & 0 & 1 \end{bmatrix}$$

$$b^i(1) = \sigma_a^i \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad b^i(2) = \sigma_a^i \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$h^i = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \qquad g^i = \sigma_m^i$$

$$\Pi = \begin{bmatrix} 1 - T_s/\tau_1 & T_s/\tau_1 \\ T_s/\tau_2 & 1 - T_s/\tau_2 \end{bmatrix}$$

where σ_a^i represents the standard deviation of acceleration noise and σ_m^i represents the standard deviation of the measurement error. For simplicity we consider the situation of similar targets only; i.e. $\sigma_a^i = \sigma_a$, $\sigma_m^i = \sigma_m$, $P_{\text{d}}^i = P_{\text{d}}$. With this, the scenario parameters are P_{d}, λ, d, V_{initial}, T_s, σ_m, σ_a, τ_1, τ_2, and the gate size γ. We used fixed parameters $\sigma_m = 30$, $\sigma_a = 0.5$, $\tau_1 = 500$, $\tau_2 = 50$, and $\gamma = 25$. Table 1 gives the other scenario parameter values that are being used for the Monte Carlo simulations.

Table 1: Scenario parameter values.[1]

Scenario	P_{d}	λ	d	V_{initial}	T_s
1	1	0	Variable	7.5	10
2	1	0.001	Variable	7.5	10
3	0.9	0	Variable	7.5	10
4	0.9	0.001	Variable	7.5	10

During our simulations we counted track i "O.K.", if

$$| h^i \hat{x}_T^i - h^i x_T^i | \leq 9\sigma_m$$

and we counted track $i \neq j$ "Swapped", if

$$| h^i \hat{x}_T^i - h^j x_T^j | \leq 9\sigma_m$$

Furthermore, two tracks $i \neq j$ are counted "Coalescing" at scan t, if

$$| h^i \hat{x}_t^i - h^j \hat{x}_t^j | \leq \sigma_m \wedge | h^i x_t^i - h^j x_t^j | > \sigma_m$$

[1]IMMPDA's $\lambda = 0.00001$ for scenarios 1 and 3

For each of the scenarios Monte Carlo simulations containing 100 runs have been performed for each of the tracking filters. To make the comparisons more meaningful, for all tracking mechanisms the same random number streams were used. The results of the Monte Carlo simulations for the four scenarios are shown in tables and figures as follows:

- The percentage of Both tracks ”O.K.”, see Table 2, and figures 2a, 3a and 4a.
- The percentage of Both tracks ”O.K.” or ”Swapped”, see Table 3, and figures 2b, 3b and 4b.
- The average number of ”coalescing” scans, see Table 4, and figures 2c, 3c and 4c.
- The average CPU time per scan (in seconds), see Table 5.

Table 2: Average % Both tracks ”O.K.”.

Scenario	IMMPDA	IMMJPDA	IMMJPDA*	JIMMPDAP
1	19	66	73	75
2	10	56	68	70
3	6	63	69	72
4	4	41	50	57

Table 3: Average % Both tracks ”O.K.” or ”swapped”.

Scenario	IMMPDA	IMMJPDA	IMMJPDA*	JIMMPDAP
1	28.3	99.96	100	96.2
2	18.9	92.5	96.8	94.6
3	8.5	99.8	100	95.8
4	5.6	76.6	80.96	82.3

Table 4: Average number of coalescing scans.

Scenario	IMMPDA	IMMJPDA	IMMJPDA*	JIMMPDAP
1	9.7	1.5	0.4	1.3
2	11.0	2.1	0.3	1.4
3	18.9	1.7	0.5	1.3
4	14.5	2.6	0.5	1.5

For the example considered, the averages in Tables 2, 3, and 4 show that IMMPDA performs less good than all others. In contrast with this, the JIMMPDA Particle filter (JIMMPDAP) outperforms the other filter algorithms when it comes to ”Both tracks O.K.”. Nevertheless, IMMJPDA* performs best regarding the ”both tracks O.K. or swapped” criterion on scenarios 1-3 and best on track coalescence avoidance for all scenarios.

Table 5 indicates a significant CPU-time increase for JIMMPDA Particle filter relative to the others. The increase is one order of magnitude for scenarios without clutter and two orders of magnitude for scenarios with clutter.

Table 5: Average CPU time per scan (in seconds).

Scenario	IMMPDA	IMMJPDA	IMMJPDA*	JIMMPDAP
1	0.016	0.022	0.023	0.439
2	0.038	0.054	0.048	7.959
3	0.014	0.020	0.020	0.438
4	0.038	0.061	0.056	7.810

6 Conclusion

In this paper we studied a Joint multi-target version of IMM and PDA. The approach taken is to first characterise the problem in terms of filtering for a jump linear descriptor system with both Markovian and i.i.d. coefficients. Subsequently exact Bayesian filter equations have been characterized. Based on these exact equations a JIMMPDA Particle filter is developed. Through Monte Carlo simulations for a simple example the JIMMPDA Particle filter algorithm has been compared to the IMMJPDA of Chen & Tugnait (2001) and the IMMJPDA* of Blom & Bloem (2002a, 2002b). All together the JIMMPDA Particle filter appears to perform best for this example, and in particular when there is clutter and missed detections. If performance is measured in track coalescence avoiding power or if CPU load is an issue, however, then IMMJPDA* is the best on this example.

References

[1] Avitzour, D., “A stochastic simulation Bayesian approach to multitarget tracking,” *Proc. IEE Radar, Sonar and Navigation*, Vol. 142, pp. 41-44, 1995.

[2] Blom, H. A. P., “An efficient filter for abruptly changing systems,” *Proc. 23th IEEE Conference on Decision and Control*, (1984), pp. 656-658.

[3] Blom, H. A. P., and Y. Bar-Shalom, “The Interacting Multiple Model algorithm for systems with Markovian switching coefficients,” *IEEE Tr. on Automatic Control*, Vol. 33 (1988), pp. 780-783.

[4] Blom, H. A. P., and E. A. Bloem, “Probabilistic Data Association Avoiding Track Coalescence,” *IEEE Tr. on Automatic Control*, Vol. 45 (2000), pp. 247-259.

[5] Blom, H. A. P., and E. A. Bloem, “Combining IMM and JPDA for tracking multiple maneuvering targets in clutter,” *Proc. 5th Int. Conf. on Information Fusion*, July 8-11, 2002, Annapolis, MD, USA, Vol. 1, pp. 705-712.

[6] Blom, H. A. P., and E. A. Bloem, “Interacting Multiple Model Joint Probabilistic Data Asscoiation avoiding track coalescence,” *Proc. IEEE Conference on Decision and Control*, December 2002, pp. 3408-3415.

[7] Chen, B., and J. K. Tugnait, "Tracking of multiple maneuvering targets in clutter using IMM/JPDA filtering and fixed-lag smoothing," *Automatica*, vol. 37, pp. 239-249, Feb. 2001.

[8] Dai, L., "Singular control systems," *Lecture notes in Control and information sciences*, Vol. 118, Springer, 1989.

[9] Doucet, A., N. J. Gordon and V. Krishnamurthy, "Particle Filters for State Estimation of Jump Markov Linear Systems," *IEEE Tr. on Signal Processing*, Vol. 49, 2001, pp. 613-624.

[10] Gordon, N. J., "A hybrid bootstrap filter for target tracking in clutter," *IEEE Tr. on Aerospace and Electronic Systems*, Vol. 33, 1997, pp. 353-358.

[11] Gordon, N. J., D. J. Salmond and A. F. M. Smith, "Novel approach to nonlinear/non-Gaussian Bayesian state estimation," *IEE Proceedings-F*, Vol. 140, pp. 107-113, 1993.

[12] McGinnity, S. and G. W. Irwin, "Multiple Model Bootstrap Filter for Maneuvering Target Tracking," *IEEE Tr. on Aerospace and Electronic Systems*, Vol. 36, 2000, pp. 1006-1012.

[13] McGinnity, S. and G. W. Irwin, "Maneuvering Target Tracking using a Multiple-Model Bootstrap Filter," Eds. A. Doucet, N. de Freitas and N. Gordon, *Sequential Monte Carlo Methods in Practice*, Springer 2001, pp. 479-497.

[14] Musso, C., N. Oudjane, and F. Le Gland, "Improving Regularised Particle Filters," Eds. A. Doucet, N. de Freitas and N. Gordon, *Sequential Monte Carlo Methods in Practice*, Springer 2001, pp. 247-271.

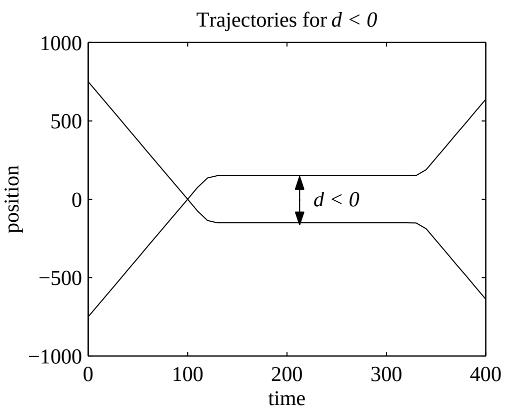
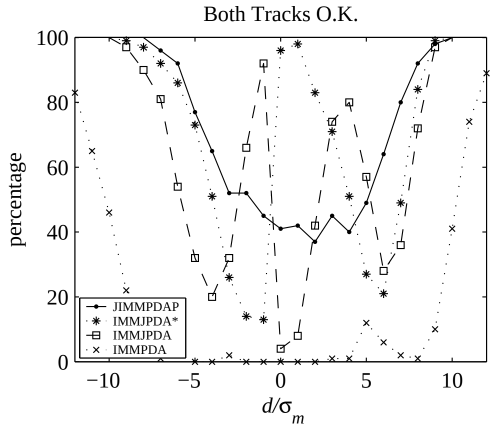
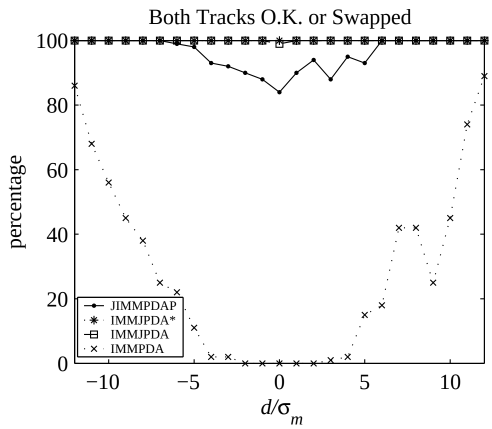


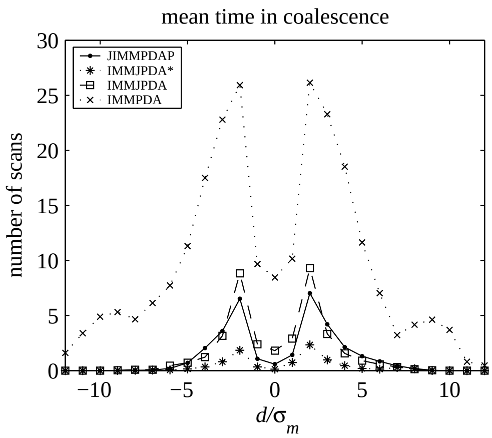
Figure 1: Trajectory example for $d < 0$



2a. Both tracks "O.K." percentage

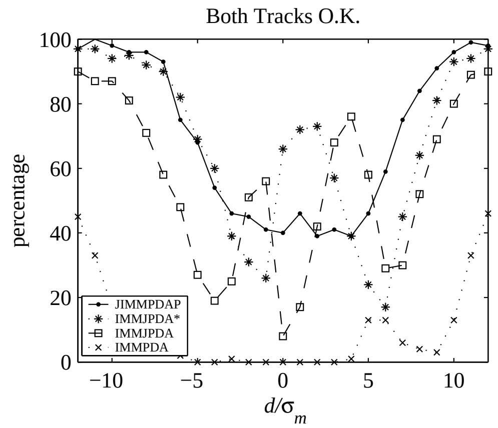


2b. Both tracks "O.K." or "Swapped" percentage

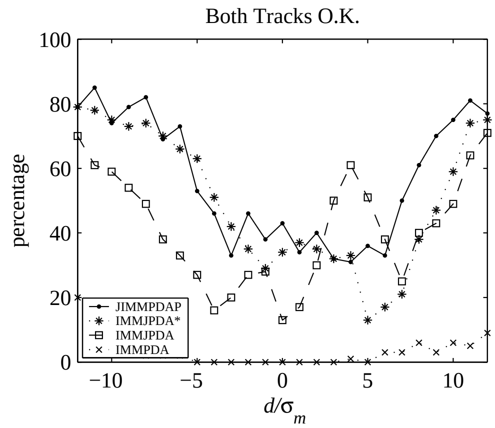


2c. Average number of "coalescing" scans

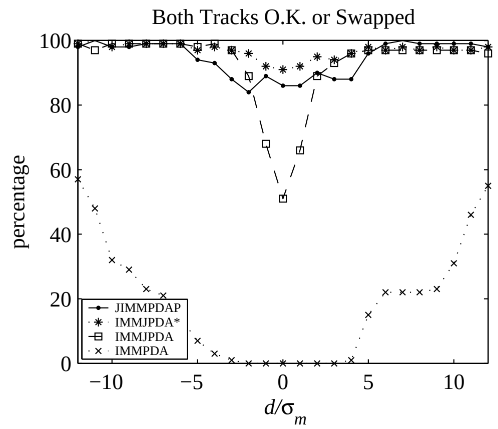
Figure 2: Simulation results for scenario 1



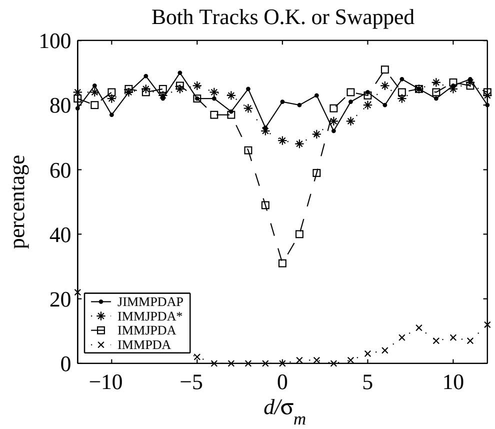
3a. Both tracks "O.K." percentage



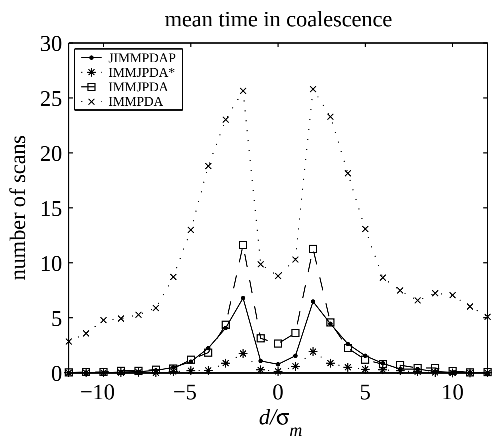
4a. Both tracks "O.K." percentage



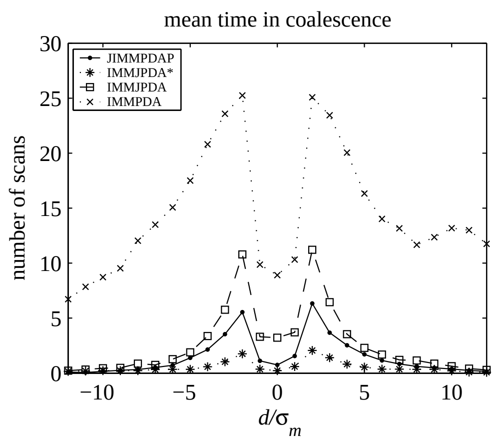
3b. Both tracks "O.K." or "Swapped" percentage



4b. Both tracks "O.K." or "Swapped" percentage



3c. Average number of "coalescing" scans



4c. Average number of "coalescing" scans

Figure 3: Simulation results for scenario 2

Figure 4: Simulation results for scenario 4